Failure Modes of Simple Pressure Vessels

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\[ \rho := 1000 \frac{\text{kg}}{\text{m}^3} \quad \text{D2P}(\text{depth}) := \rho \cdot \text{g} \cdot \text{depth} \quad \text{This function converts a depth to Pressure} \]

\[ E := 190 \text{GPa} \quad \text{Youngs Modulus} \]
\[ Y := 205 \text{MPa} \quad \text{Yield Stress} \]
\[ \nu := 0.33 \quad \text{Poisson's Ratio} \]

**Cylinder Failure Modes**

\[ \text{TD}_{\text{yield}}(\text{press}) := \frac{1}{2} \left( 1 - \sqrt{1 - \frac{2 \cdot \text{press}}{Y}} \right) \]

This function returns the Thickness to Diameter (outer) ratio of a cylinder that will cause yielding at the inner surface for a given pressure. To prevent yielding the T/D should be larger than this value. (Roark, p638 Table 32 Formula's for thick walled pressure vessel case 1c).

\[ \text{TD}_{\text{buckle}}(\text{press}) := \left[ \text{press} \cdot \left( \frac{1 - \nu^2}{2 \cdot E} \right) \right]^{\frac{1}{3}} \]

This function returns the Thickness to Diameter (outer) ratio of a cylinder for buckling. To prevent buckling T/D should be larger than this value.

Above is the composite curve for above material. To prevent failure value to Thickness/Diameter (Outer) should be greater than both those shown here for a given depth.
End Plate Failure Modes

This function calculates the Thickness to Diameter (INNER) ratio for an endplate simply supported around its edge that will cause yield failure for a given pressure applied to one side. (Roark pp428 Table 24 Formulas for flat circular plates of constant thickness Case 10a)

\[
TD_{\text{endplate}}(\text{press}) := \frac{1}{2} \sqrt{\frac{3\left(\frac{3}{v} + 1\right)}{\frac{8}{v}\text{press}}} Y
\]

To prevent failure by yielding the Thickness/Diameter (inner) ratio should be greater than that shown above. Note: This curve is for a simply supported endplate. If the edges are clamped case 10b of Table 24 in Roark should be used.

References:
Woods Hole Oceanographic Institute Technical Memorandum 3-81
Failure Curves of Cylindrical/Spherical Pressure Vessels and Flat End Caps.
By Arnold G. Sharp, August 1981

Roark's Formulas for Stress and Strain, 6th Edition
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