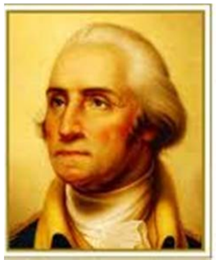

Modeling and Control of Joint Actuated Buoys

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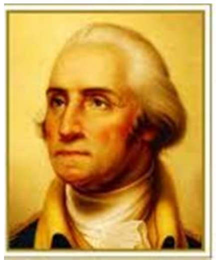
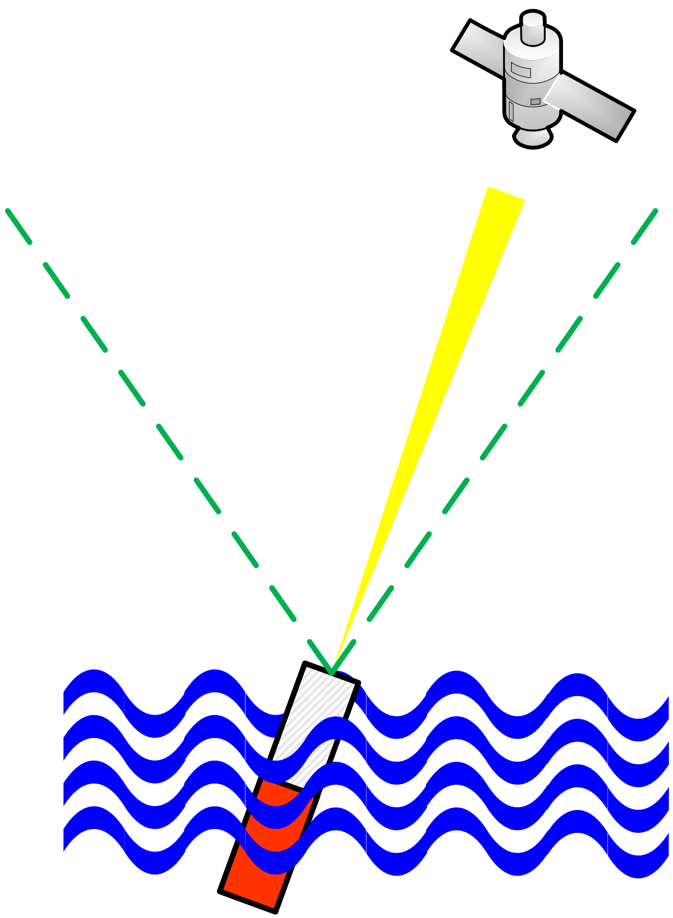


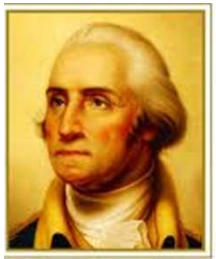
Problem Statement

Aim a payload on a small floating buoy at an arbitrary point in the sky.



For a proposed solution:

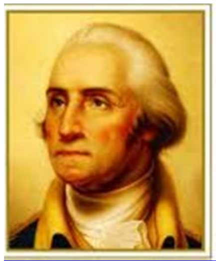
1. How much of the sky could be covered?
2. How accurately can the payload be pointed?
3. What the implications for the selection of the structure, actuators, and sensors?
4. What control laws are appropriate and effective for the system?
5. What are the critical design parameters which drive the design?



Geometry Requirements

1. The buoy is essentially a long cylinder. (launcher compatibility)
2. The payload is mechanically aimed in the desired direction. (interesting dynamics)
3. The payload makes up a “significant” portion of the buoy mass. (interesting dynamics)
4. Minimize the number of actuators. (minimize impact to payload systems)

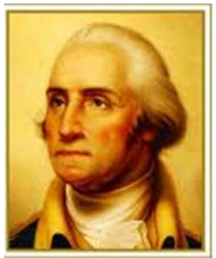
The Tyranny of the Round Tube



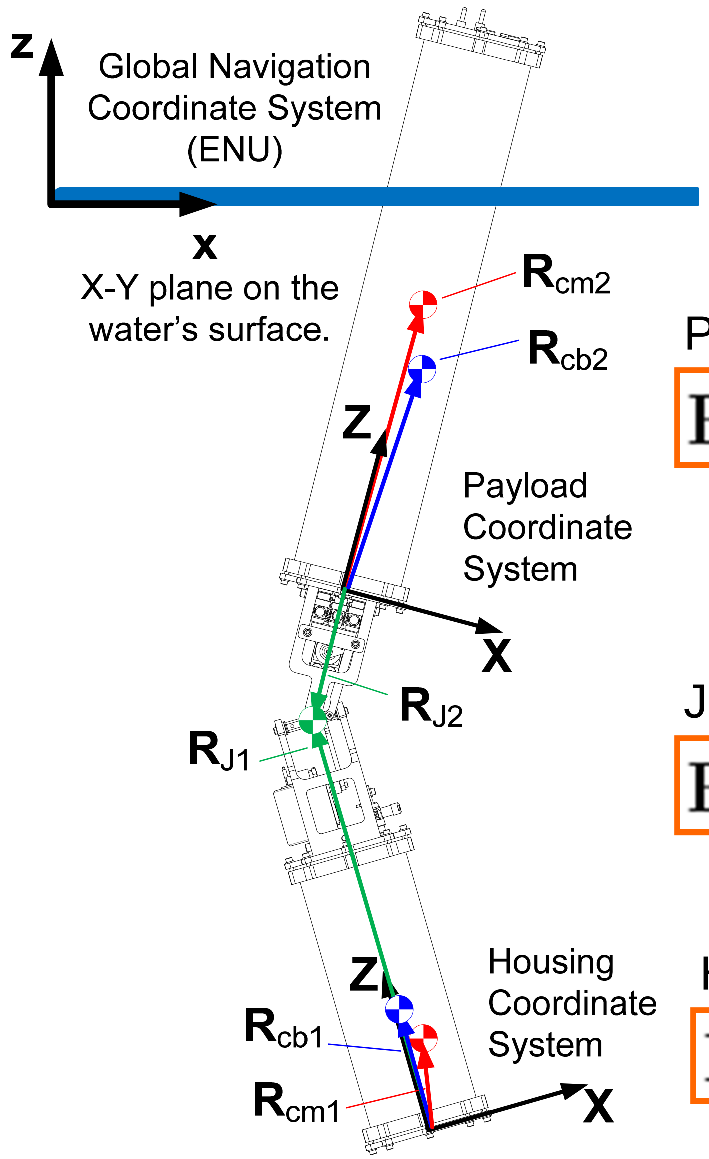
A Truth Model & Controller Development Approach

1. Pick desired primary system requirements (geometry & performance).
2. Derive, from first principles, the system dynamics to build a "truth model" in simulation.
    a) Identify which areas of the model have the most uncertainty.
    b) Devise experiments to reduce the model uncertainty (or at least bound it).
    c) Compare the tuned model with reality (quantitatively and qualitatively) as much as possible.
3. Use the "features"* of the system dynamics to inform the selection of the structure, actuators, and control schemes.
4. Use the truth model to explore the limits of the proposed solution by numerical simulation.**
5. Repeat some or all of these steps as time, money and performance permit or require.

* symmetry, preferred directions, coupling etc
** This also gives insight into the limits of the truth model.



Geometry Definitions



The payload is aimed in the direction of its Z (long) axis.

Payload Cylinder

$$\boxed{B_2,\ \Omega_2,\ \dot{\Omega}_2}$$

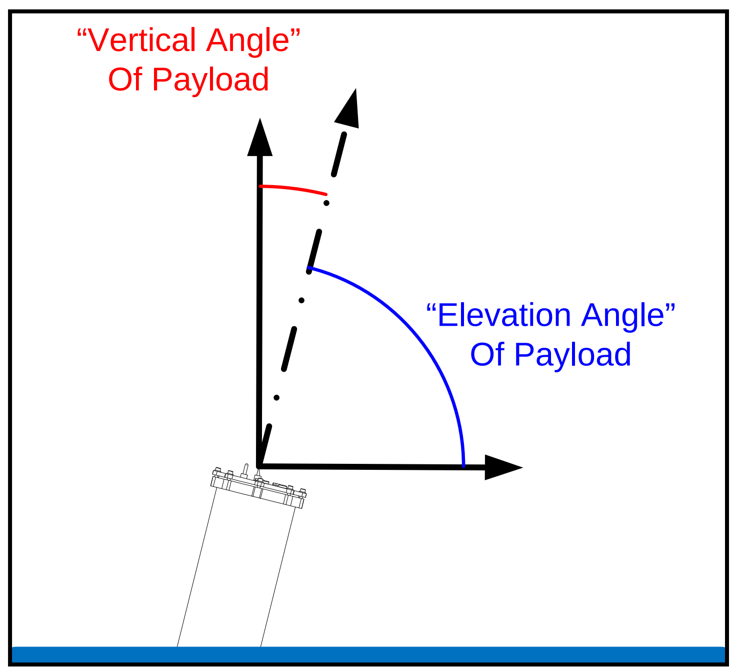
Joint

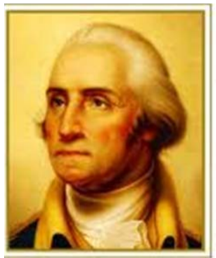
$$\boxed{B,\ \Omega,\ \dot{\Omega}}$$

Housing Cylinder

$$\boxed{B_1,\ \Omega_1,\ \dot{\Omega}_1}$$

Vertical vs. Elevation Angle





Two Body Dynamics By Direct Solution

(the really abbreviated version)

The payload's rotational acceleration is calculated **directly** from: payload PV, joint PVA, and external torques applied to each body.

The rotational system is integrated forward in time.

The housing PVA values are calculated from the payload's values

$$\begin{aligned}(\mathbf{J}_2 + \mathbf{B}^T\mathbf{J}_1\mathbf{B})\,\dot{\boldsymbol{\Omega}}_2 = \; & -\hat{\boldsymbol{\Omega}}_2\mathbf{J}_2\boldsymbol{\Omega}_2 - \mathbf{B}^T\mathbf{J}_1\mathbf{B}\left(\hat{\boldsymbol{\Omega}}\boldsymbol{\Omega}_2 - \dot{\boldsymbol{\Omega}}\right) \\ & - \left(\hat{\boldsymbol{\Omega}}_2 - \hat{\boldsymbol{\Omega}}\right)\mathbf{B}^T\mathbf{J}_1\mathbf{B}\left(\boldsymbol{\Omega}_2 - \boldsymbol{\Omega}\right) \\ & - \dot{\mathbf{J}}_2\boldsymbol{\Omega}_2 - \mathbf{B}^T\dot{\mathbf{J}}_1\mathbf{B}\left(\boldsymbol{\Omega}_2 - \boldsymbol{\Omega}\right) + \mathbf{B}^T\mathbf{T}_{ext1} + \mathbf{T}_{ext2}\end{aligned}$$

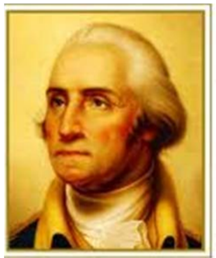
The payload rotational acceleration in terms of the payload and joint states.
The inertia tensors J_1 and J_2 are about the ***system*** CM.

The translational dynamics are calculated by Newton's second law and the rotational results.

Inertial effects are captured by the above dynamics equation.

The only external forces and moments on the model are due to: **Buoyancy**, **Gravity**, and **Drag**

"Added mass" effects are NOT included in this model.



Drag Force & Moment Approximations

These are the most uncertain approximations in model.

$$\mathbf{F}_{drag} = \begin{bmatrix} -f_{sub}C_{D_{fXY}}LR & 0 & 0 \\ 0 & -f_{sub}C_{D_{fXY}}LR & 0 \\ 0 & 0 & -C_{D_{fZ}}R^2 \end{bmatrix} \dot{\mathbf{R}}_{cm}$$

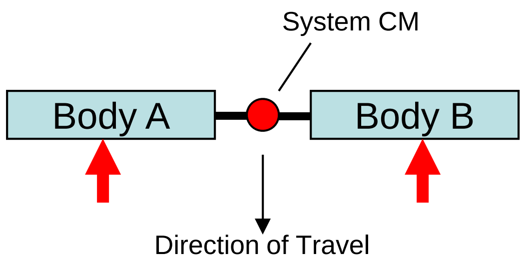
The translational drag for each body is modeled as linearly proportional to the body's velocity. The X and Y components are scaled by the submerged fraction f_{sub}.

$$M_{drag_i} = -\mathrm{sign}(\Omega_i) f_{sub} L\, R\, C_{D_{mi}} \Omega_i^2 \text{ where } i = X,\ Y,\ \text{or } Z$$

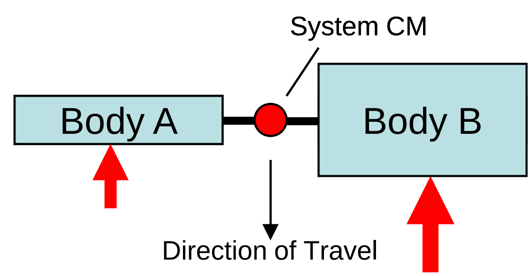
The rotational drag for each body is modeled as quadratically related to the body's velocity.

$$\mathbf{M}_{dragT} = (\mathbf{R}_{cb} - \mathbf{R}_{cmsys}) \times \mathbf{F}_{drag}$$

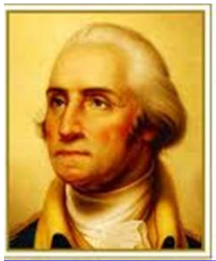
The translational drag also imparts moment about the system CM.



Body A = Body B ➔ Drag A = Drag B ➔ No Moment About System CM.



Body B > Body A ➔ Drag B > Drag A ➔ Moment about System CM.



Modeling Waves

Ref: Faltinsen and Fossen

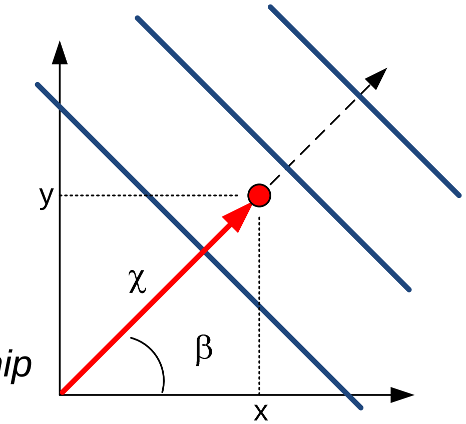
The **irregular waves** are built up from a number of components waves to estimate effects on the buoyant and drag effects.

These component waves are determined by standard spectral models of ocean waves (**Pierson-Moskowitz**).

Regular waves are modeled by N = 1.

$$k_j = \frac{w_j^2}{g}$$

The dispersion relationship for infinite water depth.



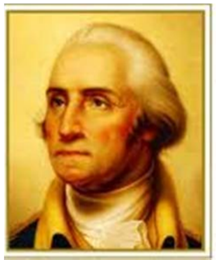
Waves propagating in the x-y plane.

Wave elevation added into buoyant effects.

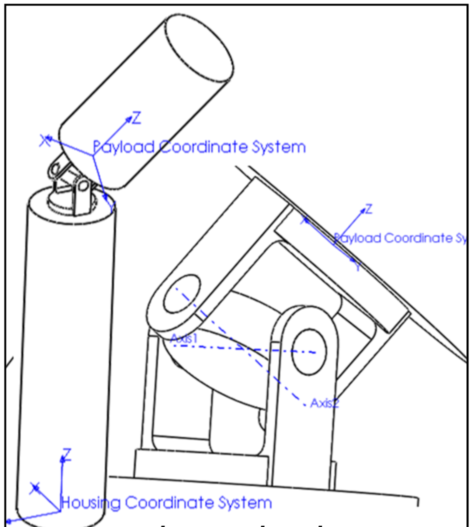
$$\zeta(x, y, t) = \sum_{j=1}^{N} A_j \sin\left[\omega_j t - k_j(x\cos\beta + y\sin\beta) + \phi_j\right]$$

Fluid velocity due to wave action added into drag effects.

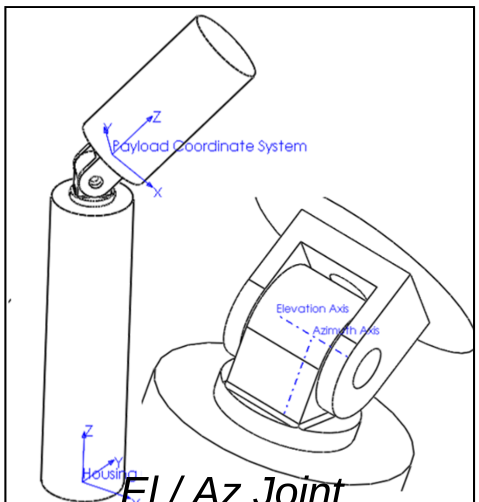
$$\mathbf{v}_{fluid}(x, y, z, t) = \sum_{j=1}^{N} \omega_j A_j e^{k_j z} \begin{bmatrix} \sin\left[\omega_j t - k_j(x\cos\beta + y\sin\beta) + \phi_j\right]\cos\beta \\ \sin\left[\omega_j t - k_j(x\cos\beta + y\sin\beta) + \phi_j\right]\sin\beta \\ \cos\left[\omega_j t - k_j(x\cos\beta + y\sin\beta) + \phi_j\right] \end{bmatrix}$$



Benefits of a Universal Joint
vs.
an Elevation Over Azimuth Joint



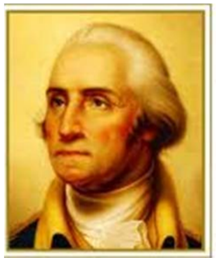
Universal Joint



El / Az Joint

1. Both axes are mechanically identical.
2. Continuous rotation in azimuth is possible **without** electrical slip-rings.
3. The system dynamics are **almost identical** for both joint axes, therefore the same control law can be used for both axes.
4. Disturbances in both roll and pitch can be immediately compensated for. (important because the buoy is equally susceptible to disturbances in both roll and pitch.)
5. The cylinder's control authority in yaw is potentially much less than roll and pitch.
   a) Roll and pitch authority are position (buoyant) driven effects.
   b) Yaw authority is a velocity and acceleration (drag & inertia) driven effect.

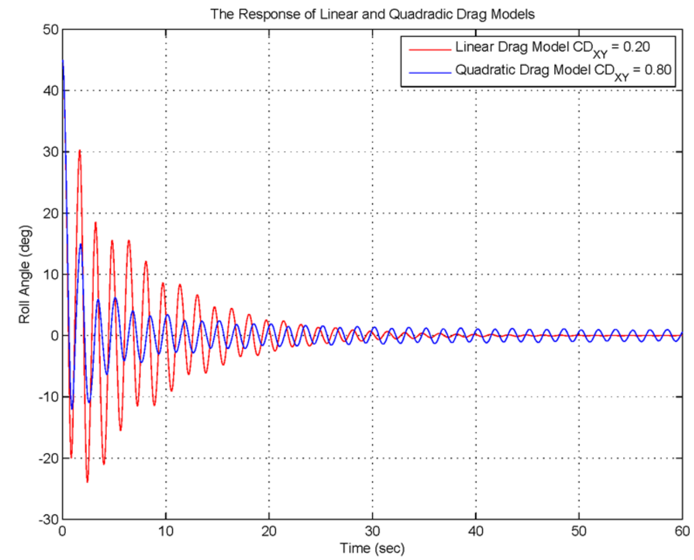
Model Validation

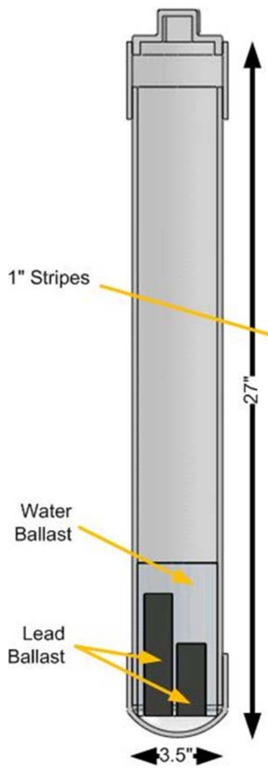


The Small Cylinder Experiment

A small non-instrumented cylinder was constructed to perform some initial validation of the numerical model.

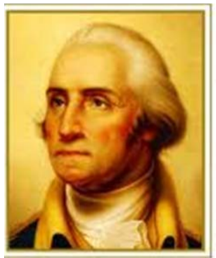
Qualitative and quantitative data was collected by reviewing the videos of the trials.





Small Cylinder Experiment Conclusions:

1. A quadratic rotational drag model is more appropriate than a linear drag model.
2. The peak of the second oscillation was significantly smaller than the initial release angle.
3. A slightly off-axis CM will produce noticeable coupling between all three rotational axes.
4. Significant cylinder motion decays away within 30 to 60 seconds of release.

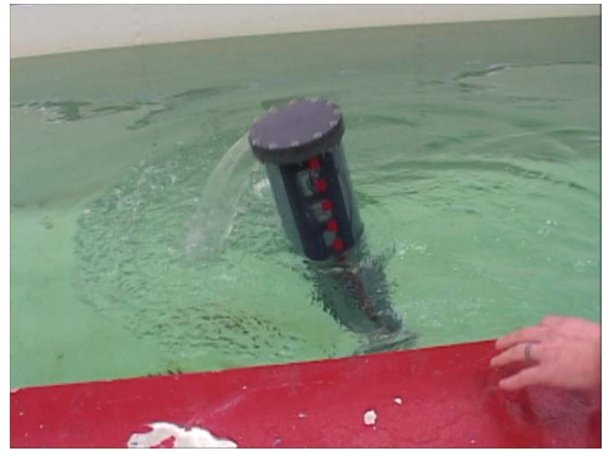


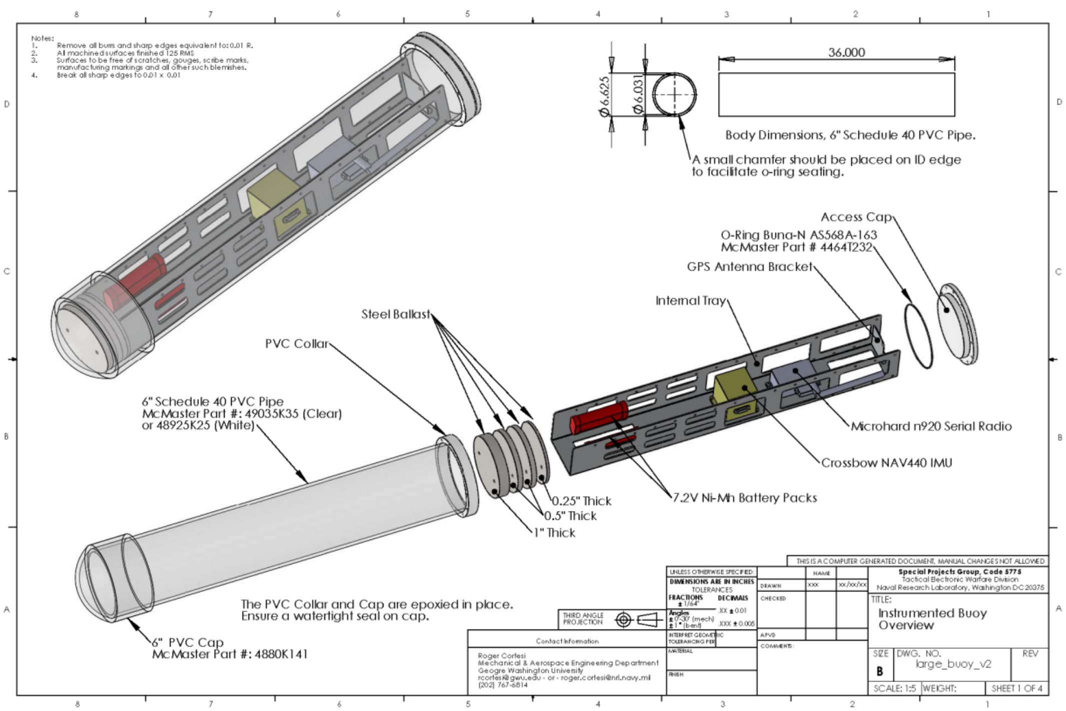
The Large Cylinder Experiments

A larger **instrumented** cylinder was constructed to perform additional validation of the numerical model.

Qualitative and quantitative data was collected by reviewing the videos of the trials and logging the data from the cylinder's IMU.

The cylinder was tested at the US Naval Academy's Hydrodynamics Laboratory.

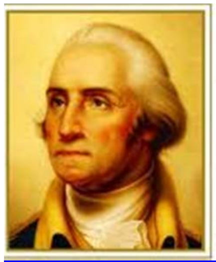




Large Cylinder Trial Results:

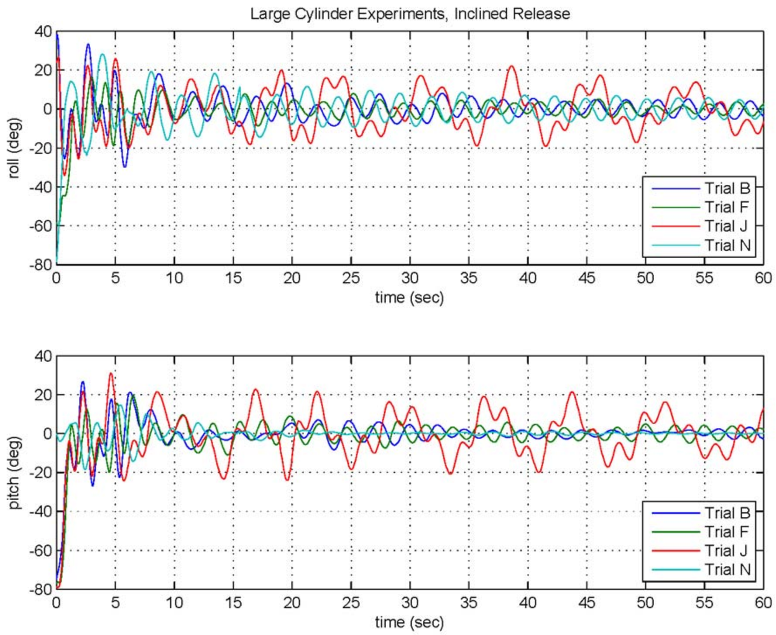
1. Translational drag forces are linear.
2. Rotational drag moments are quadratic.
3. Only slight off axis CM is required for coupling.
4. Settling times of 1 minute are reasonable.
5. Wave tank results prompted considering the resonant peak and wave velocity effects.
6. Resonant frequency estimate is accurate.

It was necessary to run a high pass filter on the IMU's roll and pitch data.



Selected Large Cylinder Plots

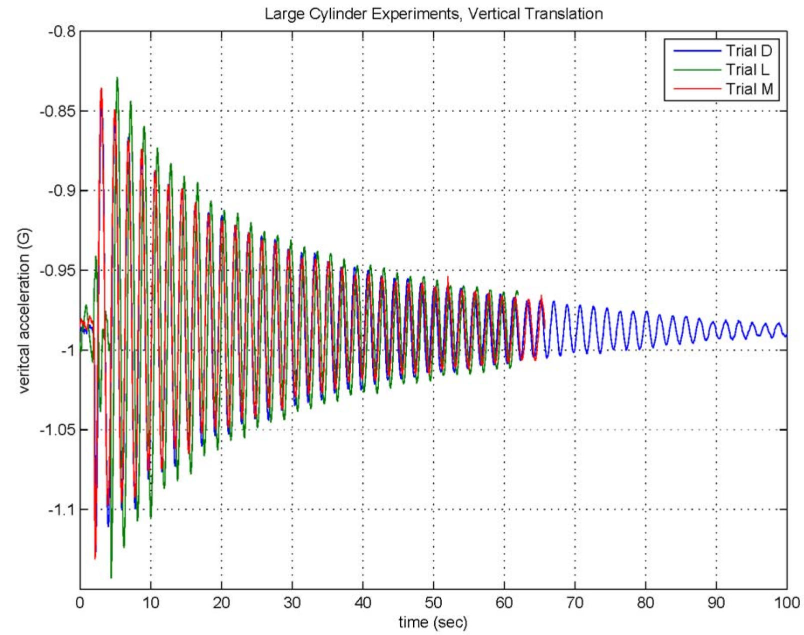
Inclined Release Trials



These trials allow tuning the following parameters in the numerical model:

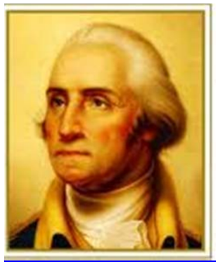
- Rotational Drag Type: Lin vs. quad] (Envelope Shape)
- Rotational Drag Magnitude (Settling Time)
- Rotational Inter-Axis Coupling

Vertical Translation Trials



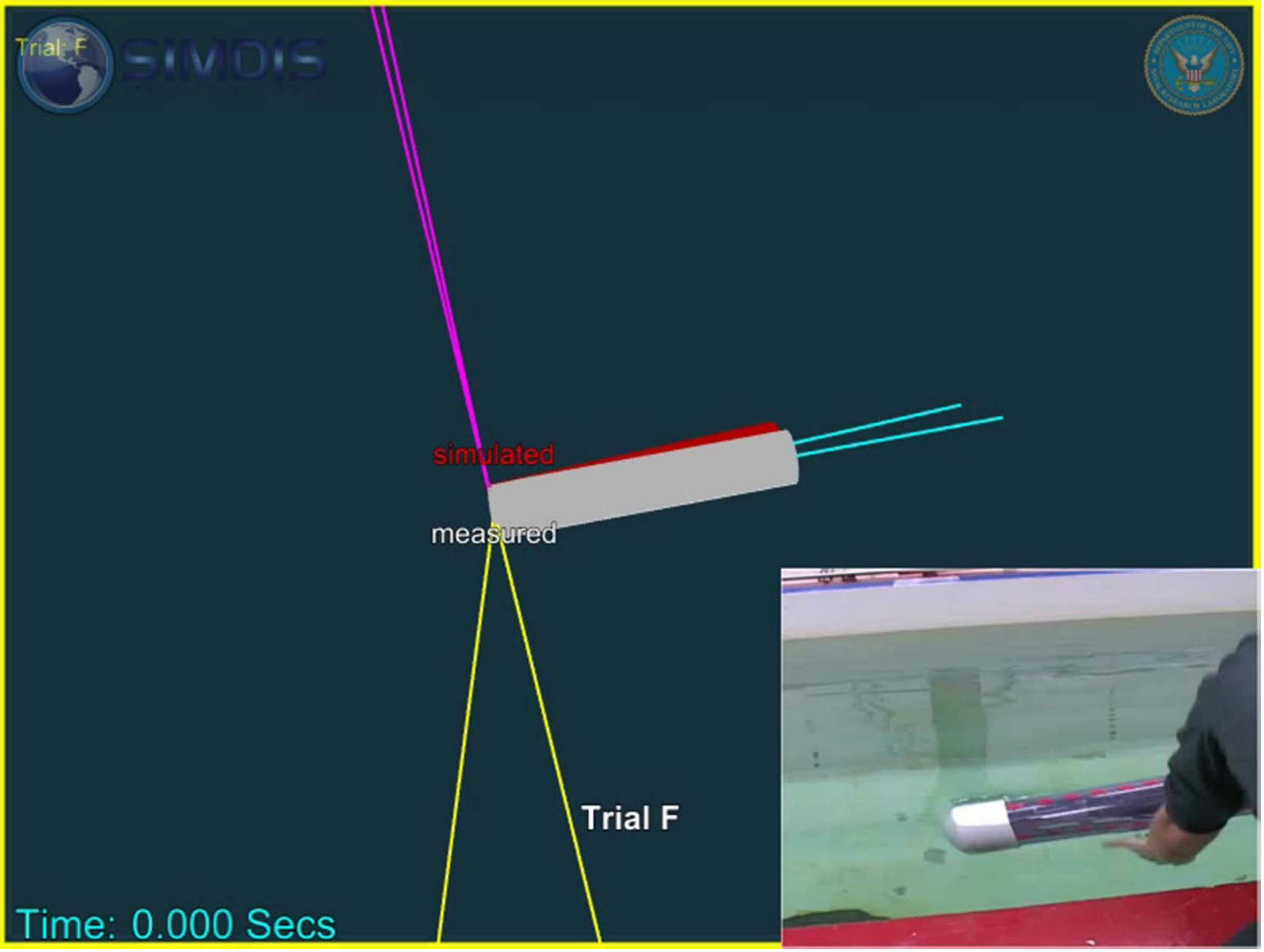
These trials allow tuning the following parameters in the numerical model:

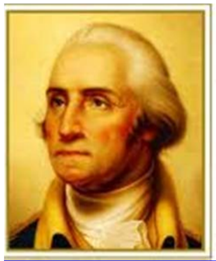
- Translational Drag Type: Lin vs. quad (Envelope Shape)
- Translational Drag Magnitude (Settling Time)
- Period of Oscillation (Peak to Peak Time)



The Large Test Cylinder Animation

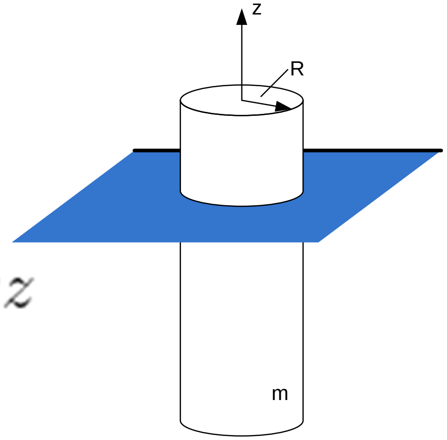
A qualitative comparison of the actual and simulated rotational response.





Resonant Frequency Validation

A cylindrical buoy's resonant frequency of vertical oscillation can accurately predicted by a simple second order model.



$$m\ddot{z} + d\dot{z} + kz = 0$$

Where the restoring force is $kz = F_{res} = A\rho g z = \pi R^2 \rho g z$

If $T_s > \dfrac{9.2}{\omega_n}$ then

$$\boxed{\omega_d \approx \omega_n = 175.46\frac{R}{\sqrt{m}} = 87.73\frac{D}{\sqrt{m}}}$$

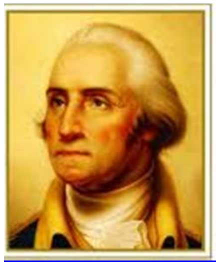
The estimated resonant frequency, where R and D are in meters, and m is in kg.

	Small Test Cylinder Experiment	**Large Test Cylinder Experiment**	**Prototype Buoy Simulation**
Estimated Period of Oscillation	1.5 sec.	1.88 sec.	1.45 sec.
Measured Period of Oscillation	1.7 sec.*	1.86 sec.	1.44 sec.**

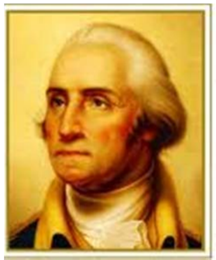
$$T_{est} = \frac{2\pi}{\omega_d}$$

* Difficult to measure accurately from the video footage. ** Calculated from the mean of the first 8 periods.

The equations above accurately predict the resonant period of vertical motion **without** using any "added mass" effects. Added mass would introduce errors of 5% to 7%.



The Control Laws



Model Linearization

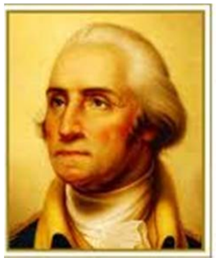
$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\\ \dot{x}_4\\ \dot{x}_5\end{bmatrix} = \begin{bmatrix}0 & 1 & 0 & 0 & 0\\ a_1 & a_2 & a_3 & a_4 & a_5\\ 0 & 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 0 & 1\\ 0 & 0 & 0 & 0 & 0\end{bmatrix} \begin{bmatrix}x_1 & \text{payload angle}\\ x_2 & \text{payload rate}\\ x_3 & \text{joint angle}\\ x_4 & \text{joint rate}\\ x_5 & \text{joint acc.}\end{bmatrix} + \begin{bmatrix}0\\0\\0\\0\\1\end{bmatrix} \begin{bmatrix}u & \text{joint jerk}\end{bmatrix}$$

The A matrix terms a_1 through a_5 specify the payload rotational acceleration as linearized functions of:

a_1: payload angle	→ payload buoyant righting moment
a_2: payload rate	→ payload drag moment (linear model)
a_3: joint angle	→ housing buoyant righting moment*
a_4: joint rate	→ housing drag moment*
a_5: joint acc.	→ housing inertia moment*

*sort of…

- All the major model characteristic appear in this model.
- The command signal is **joint jerk**.
- This model is not in regular form.
- The matrix A terms can vary a lot depending on all buoy states.
- The coupled dynamics could be modeled by making a 10x10 A matrix.



SMC Theory

Ref: Utkin and Khalil

Consider the non linear system:

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= h(x) + g(x)u \end{aligned}$$

Define the reduced order sliding manifold:

$$s(x) = a_1 x_1 + x_2 = 0, \text{ where } a_1 > 0$$

If there exists a constant k_1 such that:

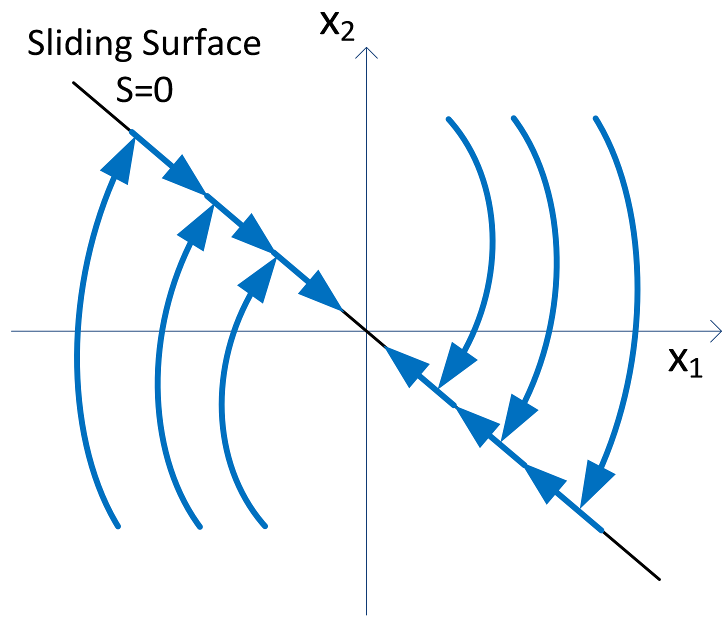
$$\text{abs}\left(\frac{a_1 x_2 + h(x)}{g(x)}\right) \leq k_1$$

Then a stabilizing control is

$$\boxed{u(x,s) = -k(x)\text{sign}(s), \ \text{for } k(x) > k_1}$$

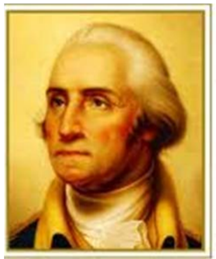
Only the **magnitudes** of the non-linearities need to be known to generate a stabilizing control.

The control law drives the system to the surface S=0 and maintains it there. **The dynamics of the system are then governed by s(x) and not h(x) and g(x).**



$$\begin{aligned} \dot{s}(x) &= a_1 \dot{x}_1 + \dot{x}_2 \\ &= a_1 x_2 + h(x) + g(x)u \\ &= a_1 x_2 + h(x) + g(x)[-k\text{sign}(s)] \\ &= g(x)\left[\frac{a_1 x_2 + h(x)}{g(x)} - k\text{sign}(s)\right] \end{aligned}$$

$$\text{sign}(\dot{s}(x)) \neq \text{sign}(s(x))$$



SMC Regular Form
Parameter Uncertainty and Disturbance Robustness

Ref: Utkin

As an example, consider the linear system

$$\dot{\mathbf{x}} = (\mathbf{A} + \Delta\mathbf{A})\,\mathbf{x} + \mathbf{B}u + \mathbf{Q}d(t)$$

$\Delta\mathbf{A}$: Parameter Uncertainty

$\mathbf{Q}$: Disturbances

If the system is in **Regular Form**, then for sliding mode control, it will be robust with respect to parameter uncertainty and external disturbances.

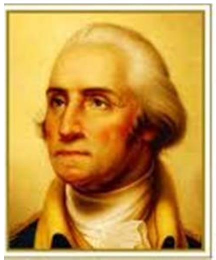
Regular Form means that $\Delta\mathbf{A}(t) \in \mathrm{range}(\mathbf{B})$ and $\mathbf{Q} \in \mathrm{range}(\mathbf{B})$

Or, equivalently, there exists matrices such that: $\boxed{\Delta\mathbf{A} = \mathbf{B}\Lambda_A \text{ and } \mathbf{Q} = \mathbf{B}\Lambda_Q}$

$$\boxed{\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ a_1 & a_2 \end{bmatrix} \begin{bmatrix} x_1 & \text{payload angle} \\ x_2 & \text{payload rate} \end{bmatrix} + \begin{bmatrix} 0 \\ b_1 \end{bmatrix} \begin{bmatrix} u & \text{joint angle} \end{bmatrix} + \begin{bmatrix} 0 \\ q_1 \end{bmatrix} d(t)}$$

This results in sliding surface of: $s = x_2 + Cx_1 = 0.$

This simpler system is in regular form (the 5x5 system was not).
The controller only needs to measure 2 states per axis (payload angle and rate).
Coupling from the other axis can be treated as a disturbance to be rejected.



Sliding Mode Control + FF Law

1) $$\begin{bmatrix} r_x \\ r_y \end{bmatrix} = f_{JxJy}(r_v,\, \phi_z) = \begin{bmatrix} \arcsin\left(\dfrac{\sin\phi_z \sin r_v}{\sqrt{1-\cos^2\phi_z \sin^2 r_v}}\right) \\ -\arcsin(\cos\phi_z \sin r_v) \end{bmatrix}$$

The **mixing equation** converts the desired azimuth and vertical angles to commanded roll and pitch angles.

---

2) For each axis, calculate the:

$$\epsilon_x = \theta_x - r_x$$ Error Signal

$$s_x = \Omega_x + C\epsilon_x$$ Sliding Parameter

$$u_x = \underbrace{-\left(\alpha \left\| \begin{bmatrix} \Omega_x \\ \epsilon_x \end{bmatrix} \right\| + \delta\right) \mathrm{sat}(s_x, h)}_{\text{Sliding Mode Term}} + \underbrace{K_{ff} r_x}_{\text{Feed Forward Term}} - \underbrace{K_i \int \epsilon_x \, d\tau}_{\text{Integral Term}}$$

Desired Joint Axis Angle

The sliding mode term is for disturbance rejection.
The feed forward term is for gross positioning of the payload.
The integral term is not used in the spatial case.

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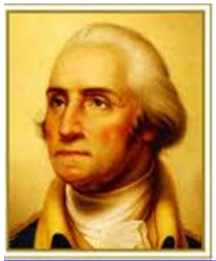
3) Then the desired joint angle is adjusted by an optional active yaw damper and soft limits are applied.



PID+FF Control Law

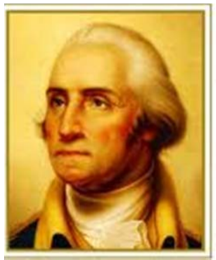
The PID+FF law was used to benchmark the SMC+FF law.

The same mixing step occurs.

The same optional yaw damper and soft limit step occurs.

$$u_x = \underbrace{-K_p \epsilon_x}_{\text{Proportional Term}} \underbrace{- K_d \Omega_x}_{\text{Derivative Term}} \underbrace{- K_i \int \epsilon_x \, d\tau}_{\text{Integral Term}} \underbrace{+ K_{ff} r_x}_{\text{Feed Forward Term}}$$

As with the SMC law, the integral gain was set to zero.



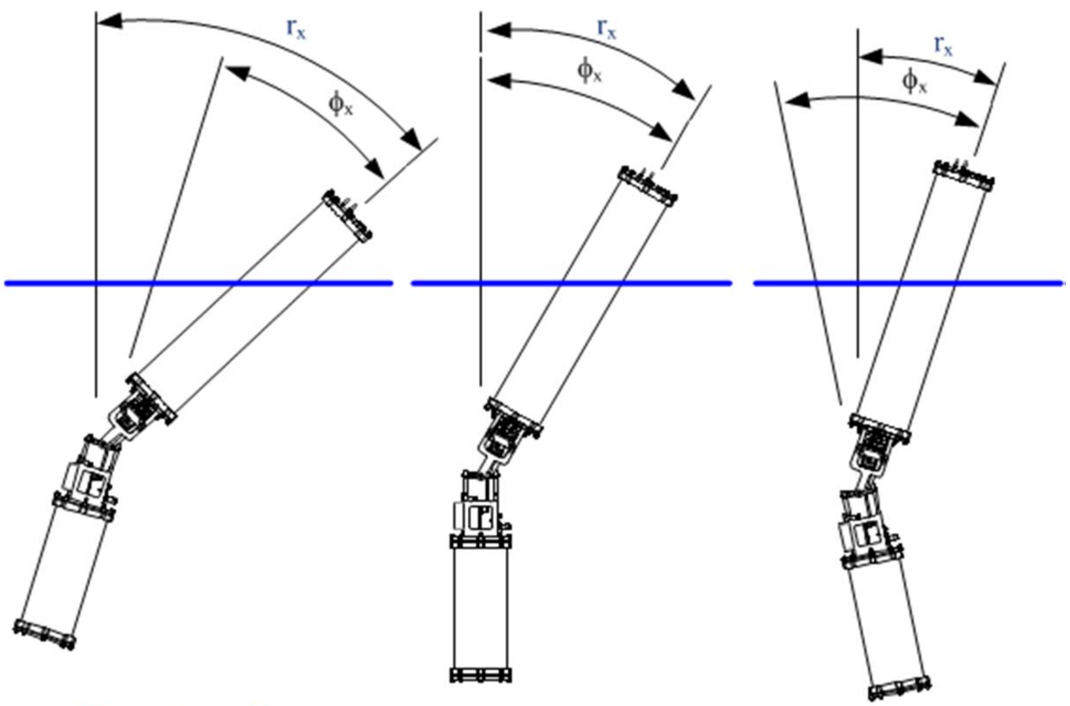
The Feed Forward Gain

The feed forward gain is one of the **critical** parameters for good controller performance.

It is determined by the mass distribution and joint location.

It can get the payload very close to the commanded direction in **static** equilibrium.

This allows the SMC or PID terms to handle only the transient response / disturbance rejection needs of the controller.



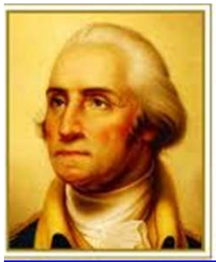
$K_{ff} < 1 \Rightarrow \phi_x < r_x$ $K_{ff} = 1 \Rightarrow \phi_x = r_x$ $K_{ff} > 1 \Rightarrow \phi_x > r_x$

Given $\phi_x = K_{ff} r_x$

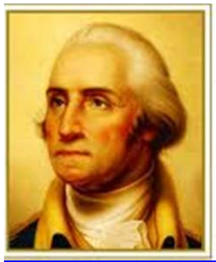
$K_{ff} < 1$: A small joint angle results in a larger payload vertical angle.

$K_{ff} > 1$: A large joint angle results in a smaller payload vertical angle.

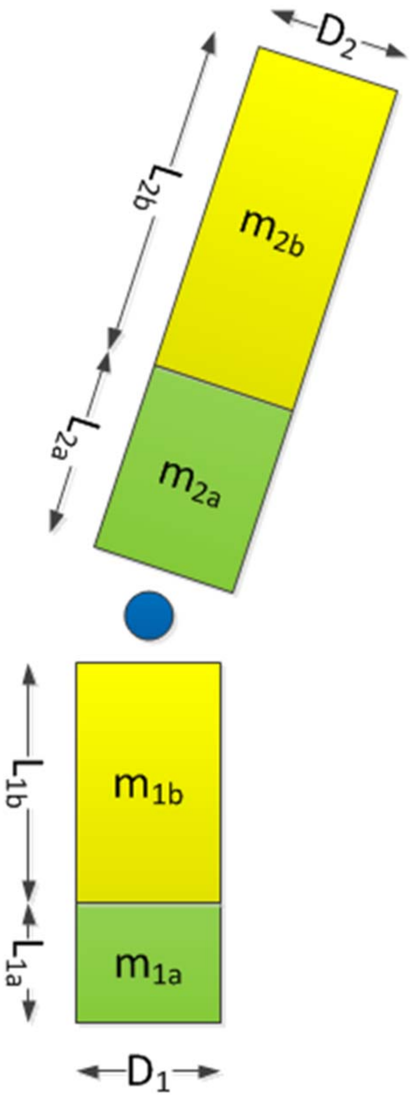
$K_{ff} = 1$: Not Possible for a floating system with a payload that has mass.



Simulation Trials



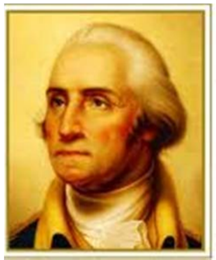
J Series Buoys



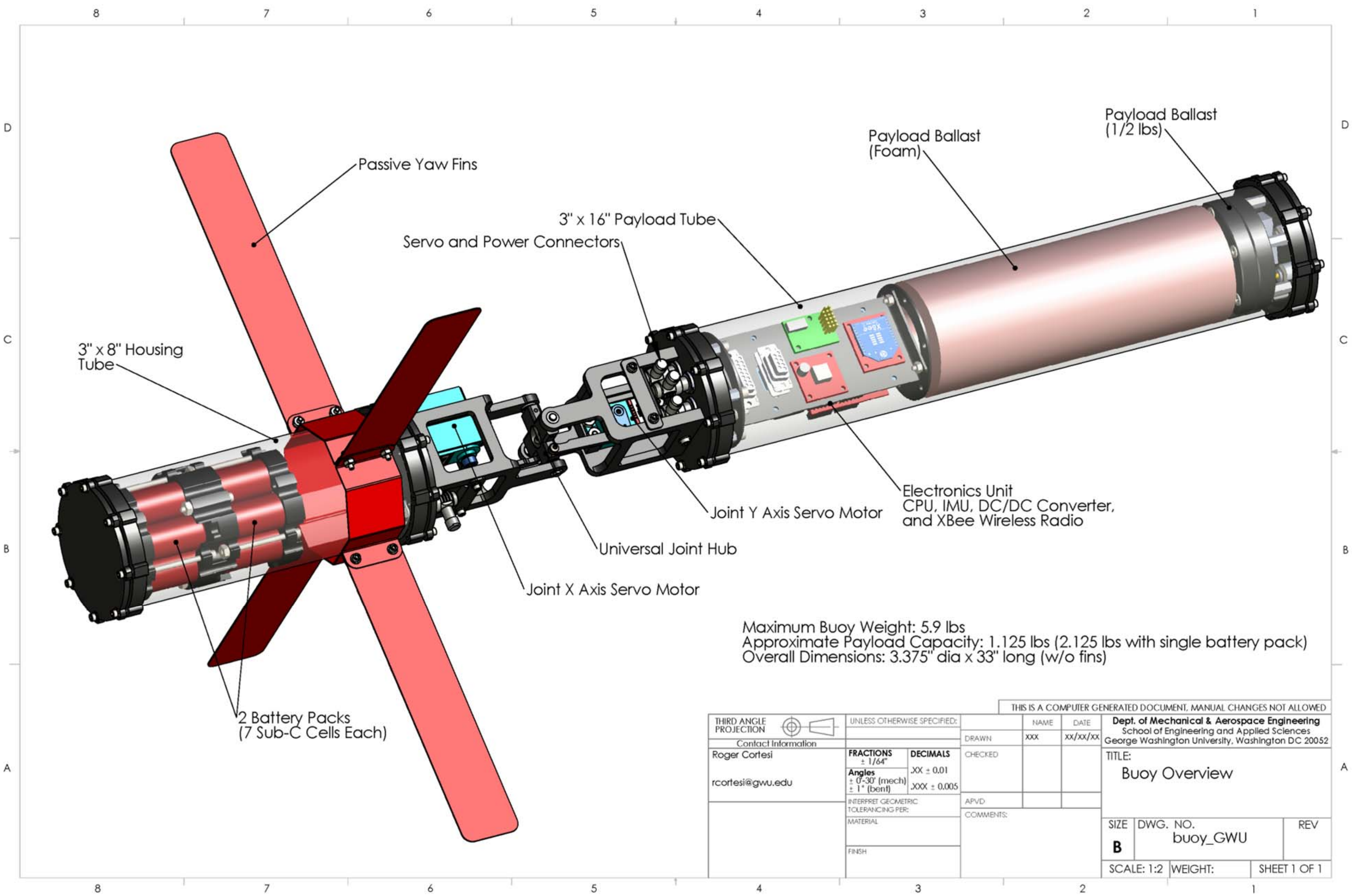
- These were a nominal buoy configuration early in the research.
- They were sized to approximate NATO size A sonobuoy.
- The housing and payload cylinders were modeled as cylinders with two homogenous regions of different densities.
- There were several variants with the same over all mass distribution but the joint in different locations.

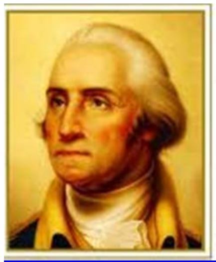
Once the truth model and control laws were tested with this rough configuration, a detailed mass model was developed to implement a configuration which could actually be built.

Resulting in the…



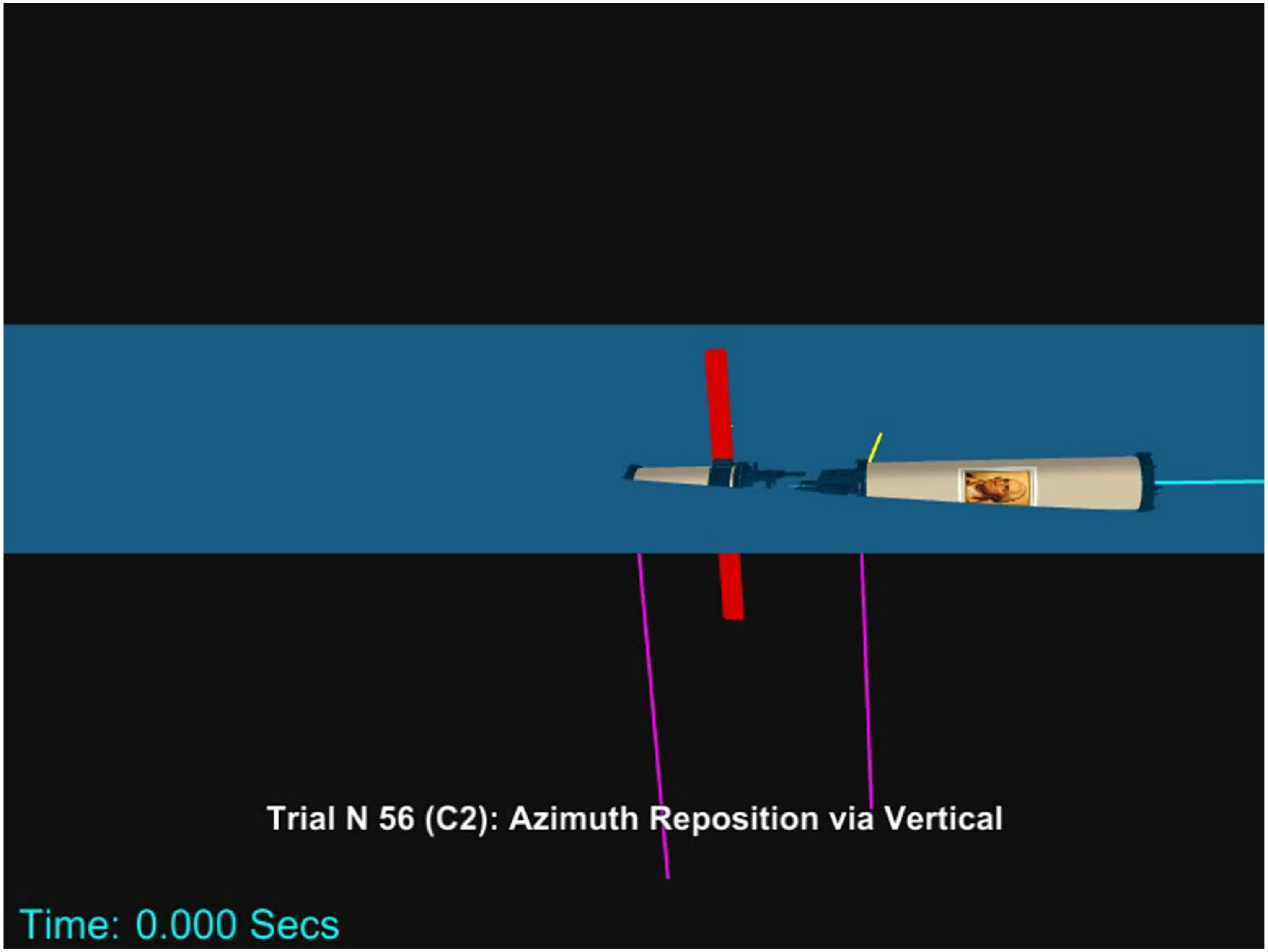
The Prototype Buoy Configuration

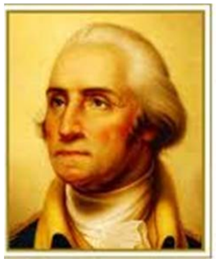




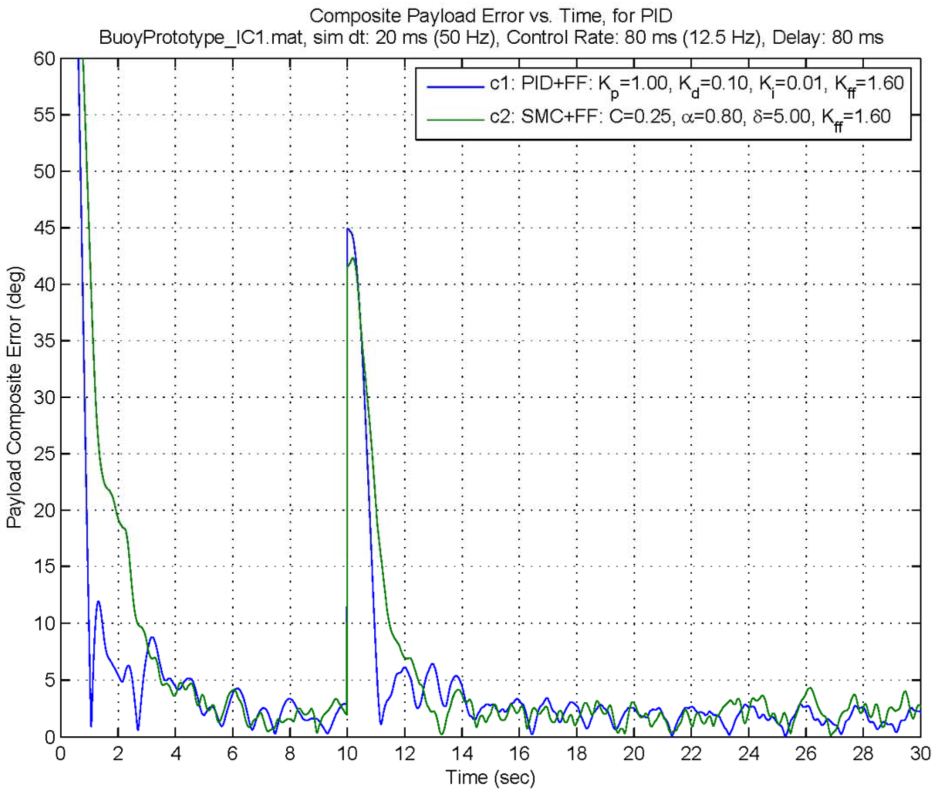
Animated Results of a Simulation Trial

No Waves, Payload Reposition in Azimuth via Vertical

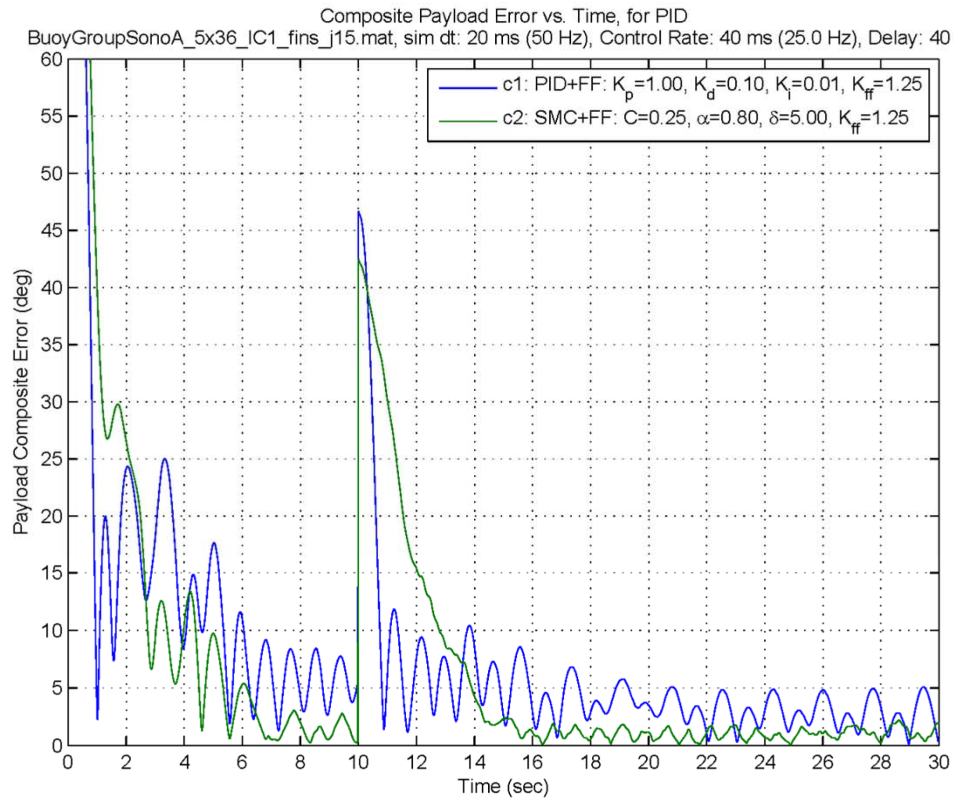




SMC vs. PID Results

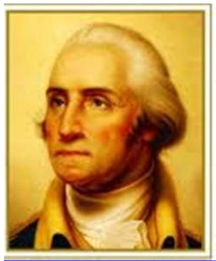


SMC and PID Controllers tuned for good performance on the prototype buoy configuration give approximately equal performance.



Apply the same controllers on a different buoy configuration (J15) changing only the K_{ff} value.

The SMC controller is less sensitive to changes in the underlying model than the PID controller.



SMC Gain Search

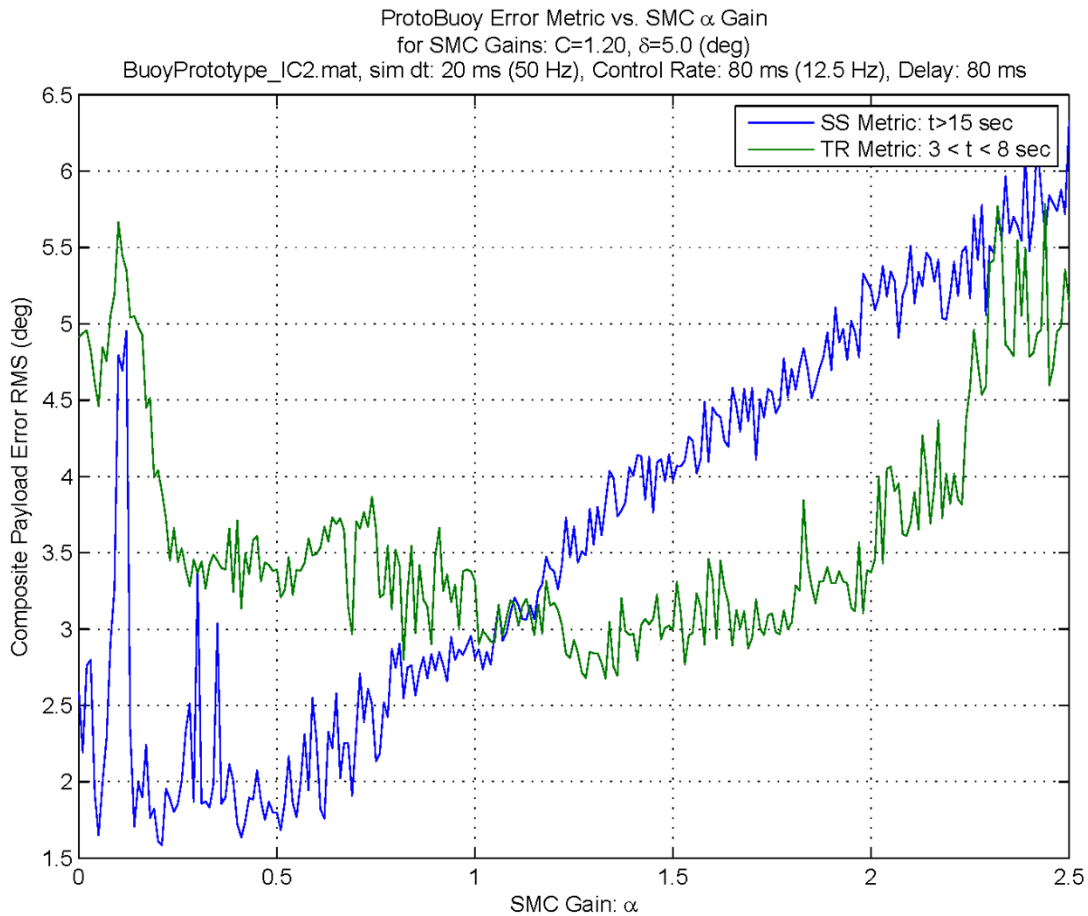
How to find the values of the SMC terms α, δ, and C which provide the "best" transient and steady state performance?

TR Performance Metric:
RMS of CPE for 3 < t < 8

SS Performance Metric:
RMS of CPE for t > 15

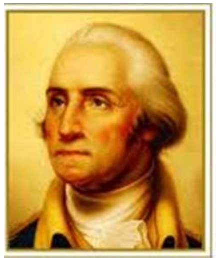
A multi-dimensional **polytope** search was **not** effective at finding good gain values due to noise on performance metrics.

Two rounds of single dimensional searches were performed with values from the first feeding the second.



Here approximately 300 trials are performed varying gain α while holding δ and C constant. The TR and SS Metrics are plotted for each trial.

Good SMC Gains for the Prototype Buoy Configuration
α=0.5, δ = 5 (deg), C = 1.5

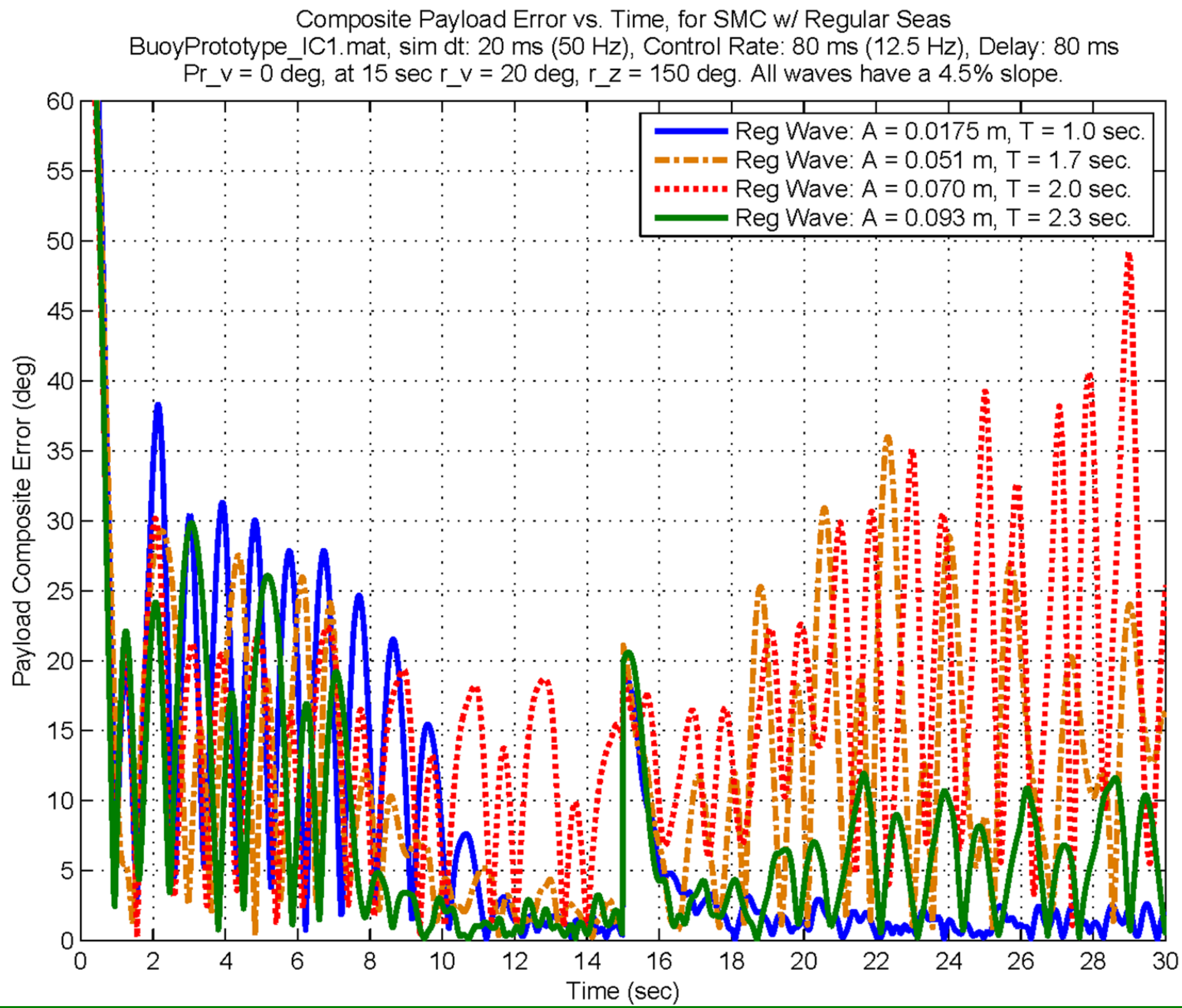


Regular Sea Effects

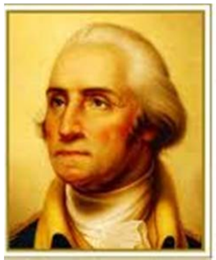
Four regulars waves, each with a slope of 4.5% are applied to the buoy.

This is a result of the **passive characteristics** of the buoy structure, **NOT** the control law.

A **tuned-mass-damper** built into the buoy could be used to address the resonant effects.



Regular wave periods of approximately 1.5 to 2.2 seconds excite a resonant response in the prototype buoy configuration.



Irregular Sea Effects

How much energy is near the resonant peak in actual ocean waves?

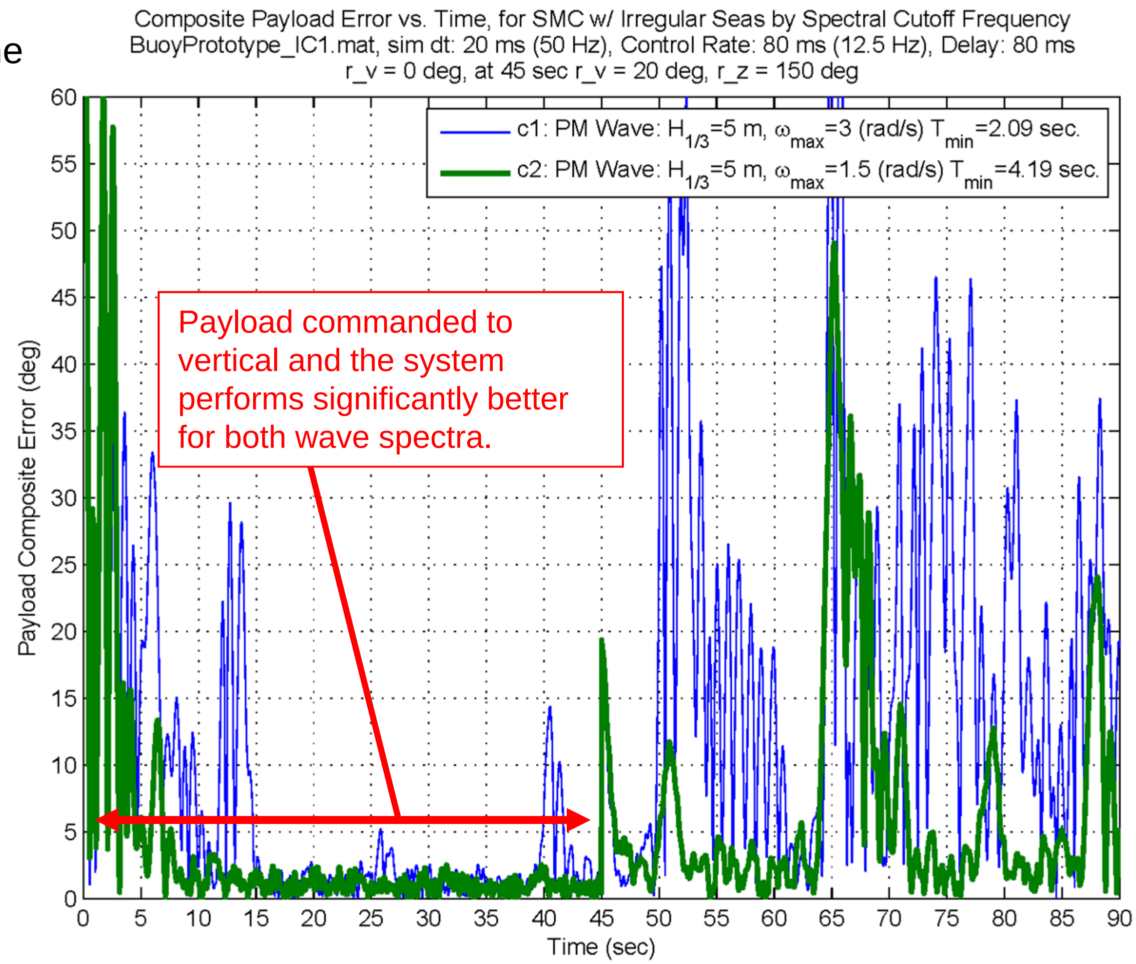
An irregular sea waveform generated using the **Pierson-Moskowitz** spectrum for $H_{1/3}$=5m.

Cutoff Frequency = 3 rad/s (T = 2.1 sec)
Cufoff Frequency = 1.5 rad/s (T = 4.2sec)

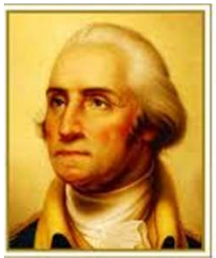
Buoy starts horizontal and the payload is commanded to vertical for the first 45 seconds, then commanded to r_v = 20°.

It is not clear from the literature if using the various standard wave spectra for frequencies greater than 1 rad/s is valid.

The high frequency content (> 1 rad/s) of actual ocean waves **is a significant element of model uncertainty**.



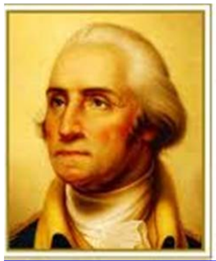
A vertically stabilized payload is significantly more robust than a non vertical payload.
The high frequency content (> 1 rad/s) of the wave spectrum **is a critical factor** in system performance.



Design Choices

(where else this could have gone)

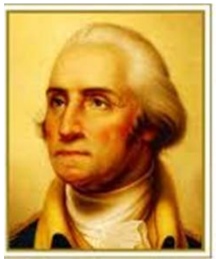
Basic Structure Type:	Inflatable Structures	Flexible Structures	Multi-Body	Single Body
Basic Structure Shape:	High Aspect Ratio Shapes (cylinder, spar, etc.)	Low Aspect Ratio Shapes (ring buoy, flat sheet, etc.)		
Passive Control Features:	Fins	Mass / Actuator Distribution	Tuned Mass Damper	
Actuator Type:	Flapping Fins	Joint Actuated	Thrusters	
Joint Type:	El. Over Az. Joint	Universal Joint	Spherical Joint	
Controller Type:	PID	SMC	LQR / LQI	
Simulation Type:	Direct Dynamics Solution	Method of Constraint Equations.		



Conclusions I

(Primary)

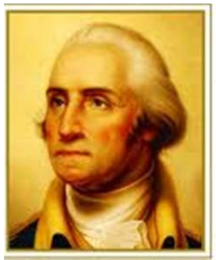
- When the **payload is stabilized about the vertical it is significantly more robust** with respect to parameter uncertainty and disturbances.
- The **feed forward term is critical** for improving system performance away from the vertical. It encodes information about the static equilibrium of the system, leaving the other terms in the control laws to handle the transient performance.
- The **resonant frequency of the structure is one of the critical parameters** in determining system performance in the presence of waves.
- The **actual frequency content of the ocean waves above 1 rad/s** is a **significant source of uncertainty** in the predicted performance using this truth model.
- Treating **coupling from the other axes as disturbances** results in significantly simpler control laws.
- Formulating the **system model in regular form** is important for SMC to be effective and **results in using fewer state variables**.
- PID and SMC control (w/ feed forward) can perform comparably.



Conclusions II

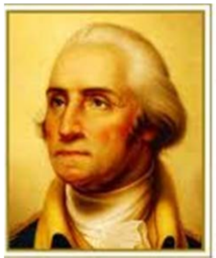
(Secondary)

- The passive (fins) and active yaw dampers may or may not be important for successful control of the buoy.
- The effects of control rate, system latency and actuator acceleration under go a step like transition with respect to system performance.
- Significant rotational coupling is introduced by having the CM only slightly off the central axis.
- Actuation chatter was reduced in the SMC law by
  - A linearized boundary layer about the sliding surface.
  - The presence of the feed forward term in the control law.
  - The low level actuator controller acts as a low pass filter.
- The drag approximations used were shown to be reasonable.



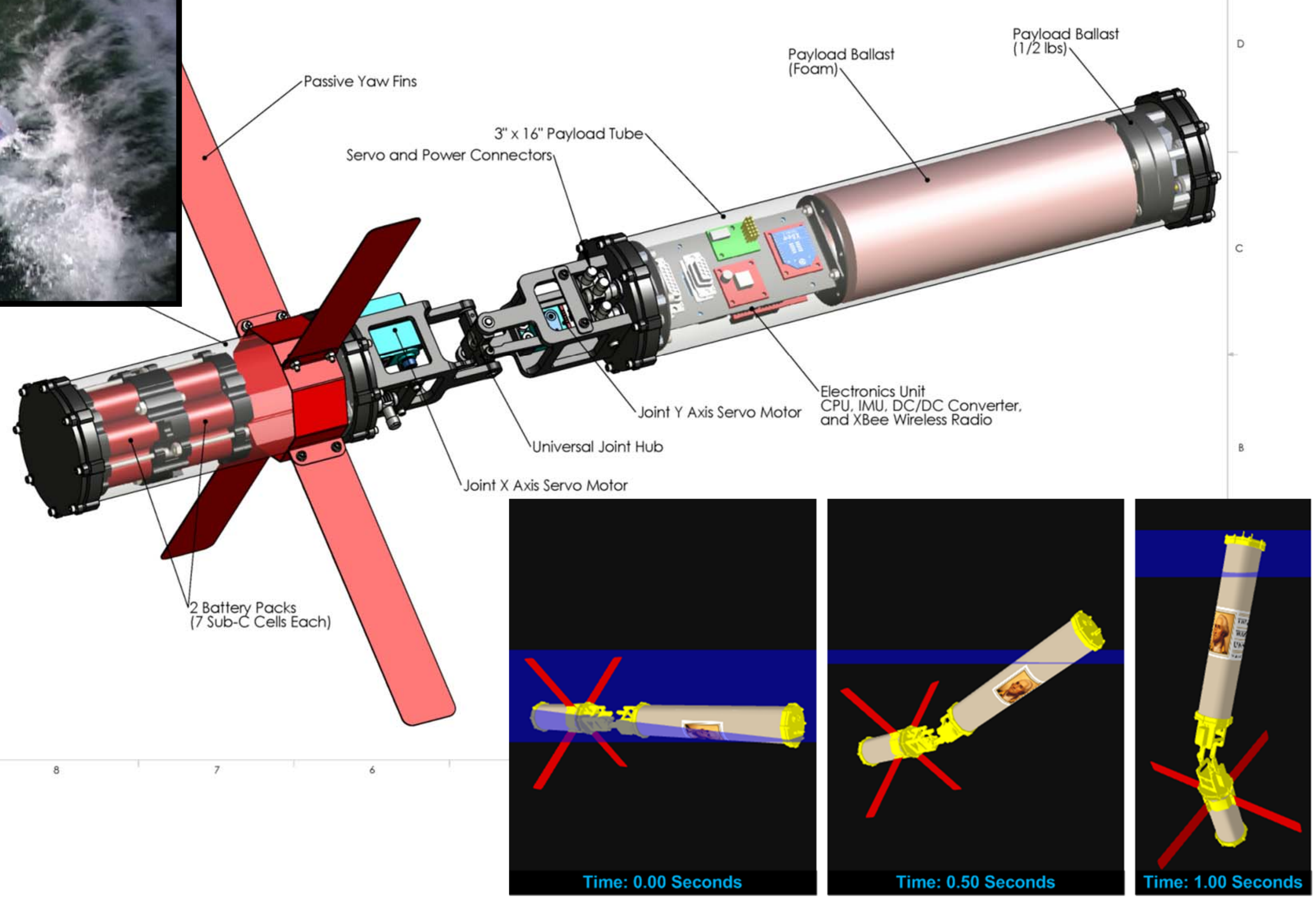
Future Work

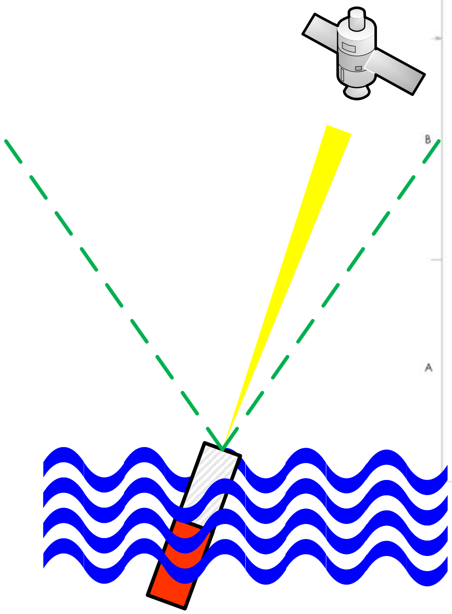
- Build and test the prototype buoy and control laws in a wave tank and at sea.
- Characterize the high frequency content of various sea states for frequencies greater than 1 rad/s.
- Investigate the use of a tuned-mass-damper to improve buoy’s resonant response.
- Investigate the use of alternate structure types (unfolding or inflatable) to improve the passive performance of the system.
- Investigate the use of alternate actuation schemes (thrusters, flapping fins, etc.).

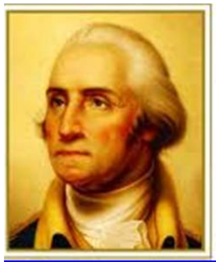


Questions

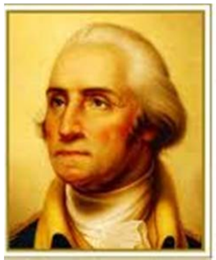
$$u_x = -\left(\alpha \left\| \begin{bmatrix} \Omega_x \\ \epsilon_x \end{bmatrix} \right\| + \delta\right) \mathrm{sat}(s_x, h) + K_{ff} r_x - K_i \int \epsilon_x \, d\tau$$







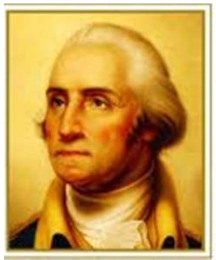
Back Up Slides



Other Simulation Results

(plots available in the backup slides)

- The prototype buoy configuration can perform well with *no waves* up to about 30-35 degrees from the vertical.
- The control rate is not the limiting factor in performance until it is slower than 10 Hz (100 milliseconds).
- The system latency is not the limiting factor in performance until it is longer than 120 milliseconds.
- The actuator acceleration is not the limiting factor in performance until it is less than 300 degrees / sec^2.
- For the prototype buoy configuration the fins only slightly improved the system performance and the active yaw damper provided no improvement.
- Two approximately similar initial conditions yielded significantly different performance. This is something to be considered in future work for model validation.

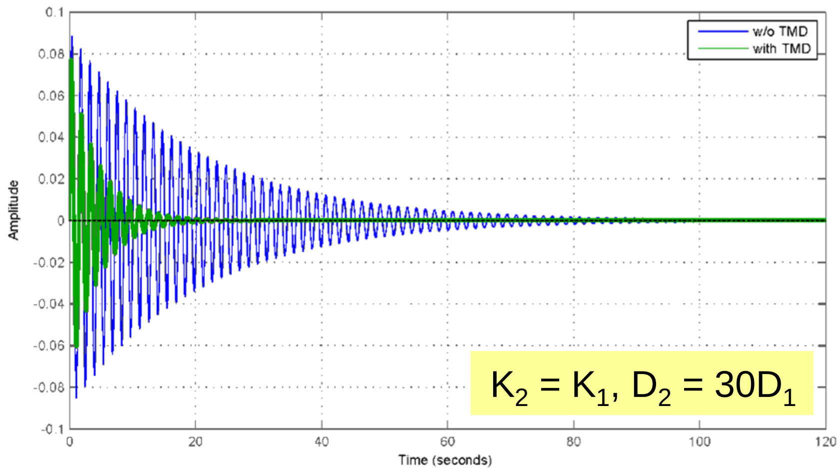


Tuned Mass Damper Results

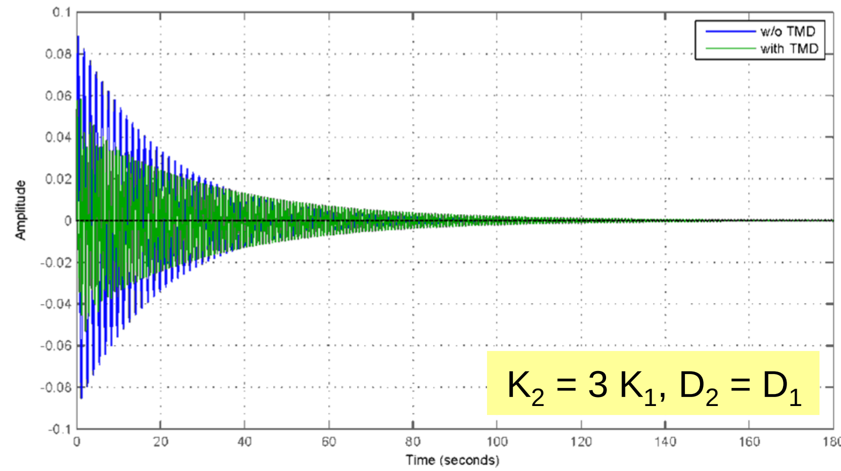
Mounting the battery on springs & dampers.
Makes it a three body system.

w/o TMD
w/ TMD

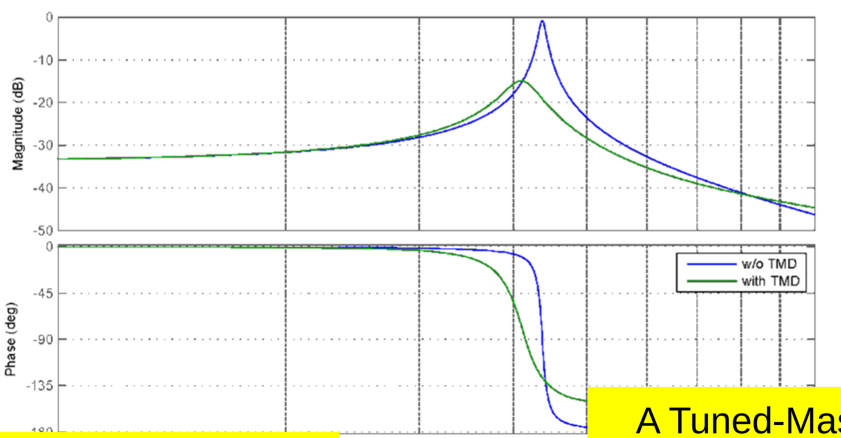
Impulse Response



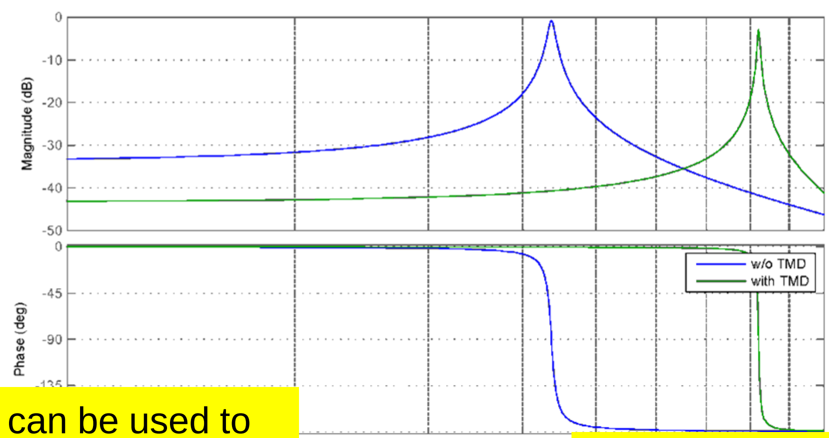
Impulse Response



Bode Plot



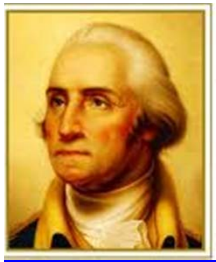
Bode Plot



Reduce Res. Peak.

A Tuned-Mass-Damper can be used to shape the resonance in two ways:

Shift Res. Peak.



Pierson-Moskowitz Wave Spectrum

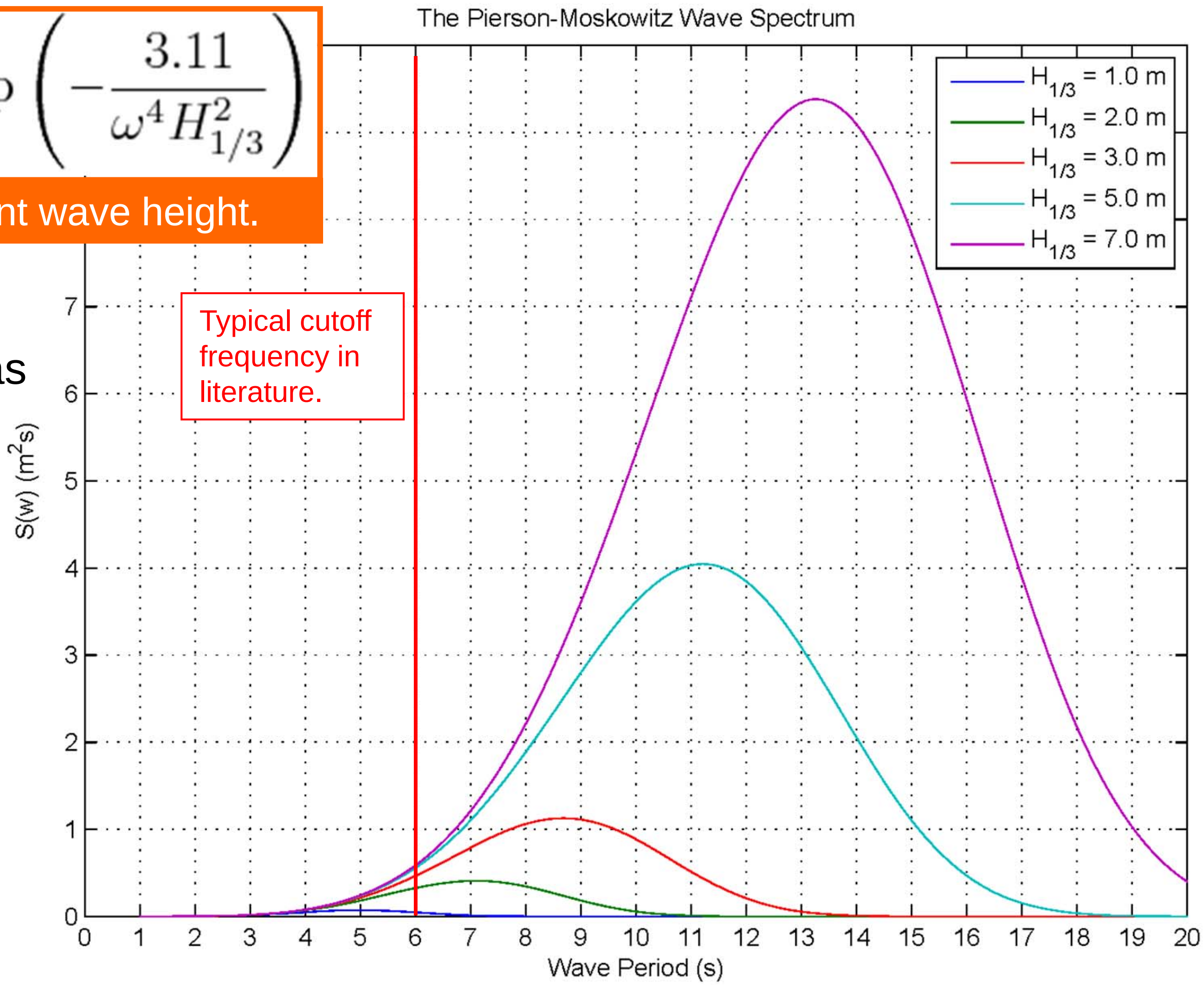
$$S(\omega) = \frac{0.0081 g^2}{\omega^5} \exp\left(-\frac{3.11}{\omega^4 H_{1/3}^2}\right)$$

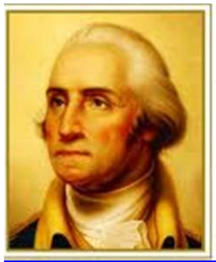
Function only of significant wave height.

Assumes:
- Fully developed seas
- Unlimited fetch
- Infinite water depth

Component wave amplitude calculated from:

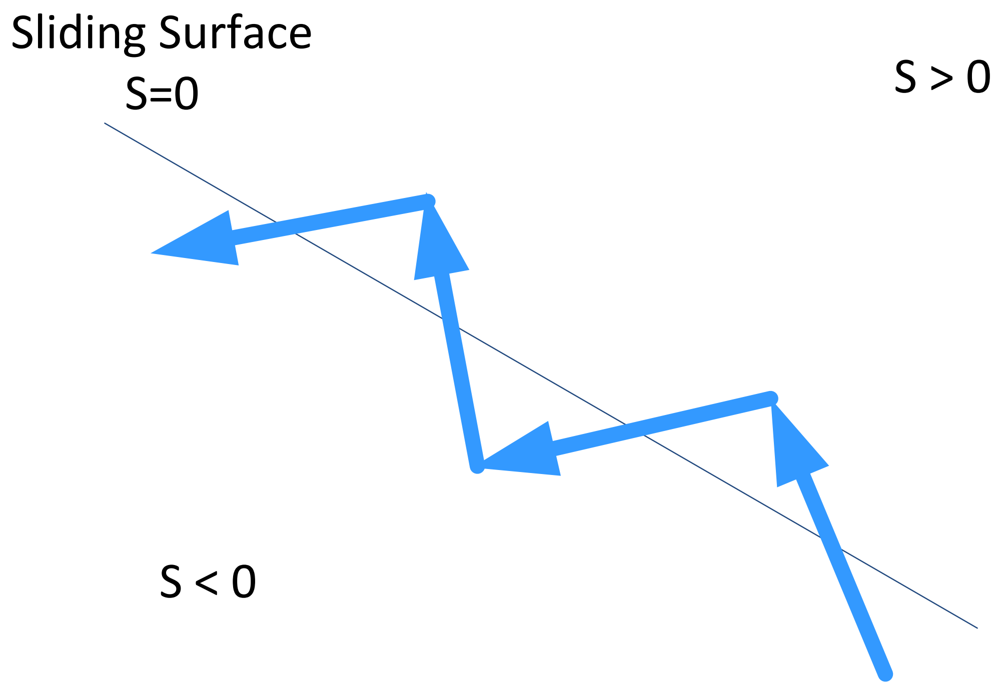
$$A_j = \sqrt{2S(\omega_j)\Delta\omega}$$





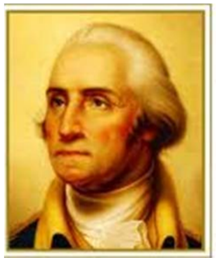
Sliding Mode Chattering

Delays and un-modeled dynamics can cause the system to oscillate about the sliding surface, causing the control to oscillate between +k and –k.

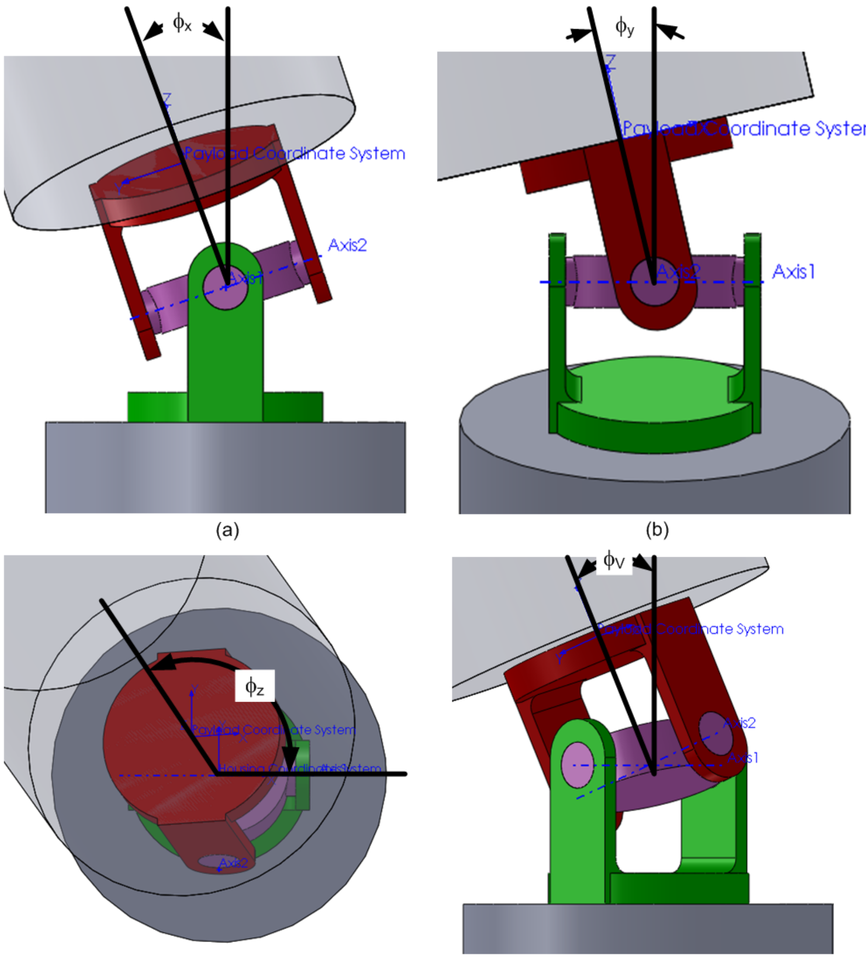


Numerous techniques are presented in the literature minimize chattering, including:

- Placing a boundary layer around the sliding surface.
- Low pass filters on actuators.
- Using the SMC to drive an observer.



Kinematics of the Universal Joint



(a) (b) (c) (d)

$$\phi_x = \arcsin\left(\frac{\sin\phi_z \sin\phi_v}{\sqrt{1-\cos^2\phi_z \sin^2\phi_v}}\right)$$

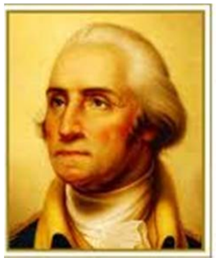
$$\phi_y = -\arcsin(\cos\phi_z \sin\phi_v)$$

The orientation of the joint can be specified by the individual axis angles,

$$\phi_z = \operatorname{atan2}(\sin\phi_x \cos\phi_y, -\sin\phi_y)$$

$$\phi_v = \arccos(\cos\phi_x \cos\phi_v)$$

or the overall azimuth and vertical angle of the joint.



More Kinematic of the Universal Joint

The joint DCM is built up from the axis DCMs.

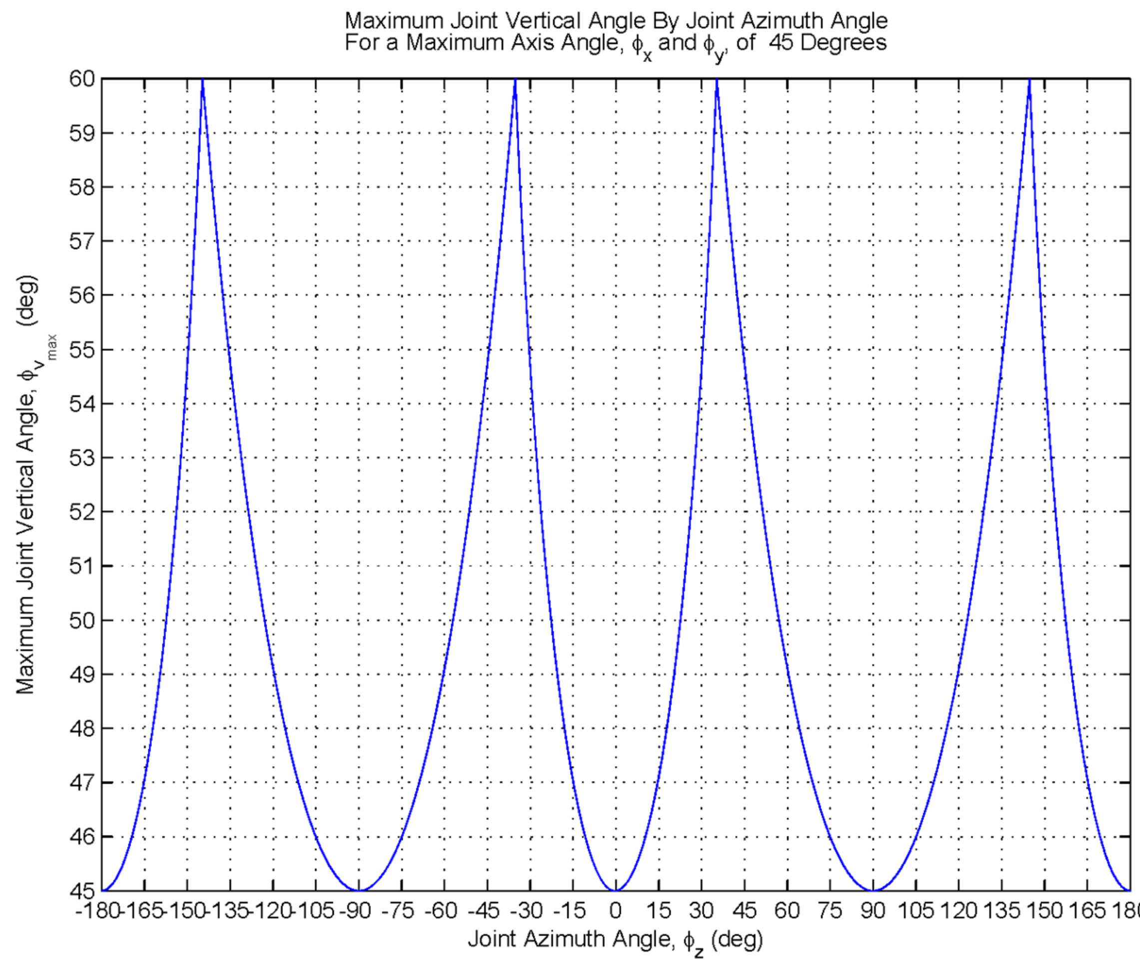
$$\mathbf{B} = \mathbf{B}_x \mathbf{B}_y$$

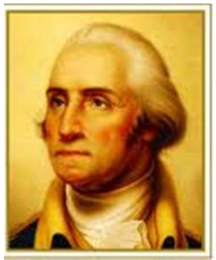
$$\mathbf{B}_x = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi_x & \sin\phi_x \\ 0 & -\sin\phi_x & cos\phi_x \end{bmatrix}$$

$$\mathbf{B}_y = \begin{bmatrix} \cos\phi_x & 0 & -\sin\phi_y \\ 0 & 1 & 0 \\ \sin\phi_y & 0 & cos\phi_y \end{bmatrix}$$

The orientation of the payload and housing with respect to the joint axis determines the order of multiplication. It is *not* commutative.

Maximum vertical angle varies as a function joint azimuth angle.





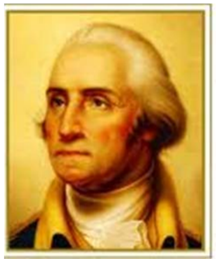
Added Mass Effects

Added mass effects are NOT included in the current simulation model and results.

Added mass would be much more “cleanly” handled in a simulation framework that uses constraint equations, rather than the direct dynamics solution.

Added mass effects the buoy system in the following ways:

- Added mass could lower the resonant frequency of the buoy. This was seen in the data.
- Added mass could slow the horizontal translation of the system. These are not important states for the system.
- Added mass could make the yaw fins more effective. By not including it the results are more conservative.
- Added mass might result in more actuator torque required to move the joint.



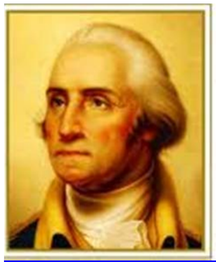
Translational Dynamics

With the position and the velocity of the joint known and the attitude and rotational rates of both bodies known, the position and velocities of all the other points can be computed.

$$
\begin{aligned}
(m_1 + m_2)\,\ddot{\mathbf{d}}_0 = \; & -m_1 \left(\dot{\mathbf{B}}_1 \hat{\Omega}_1 + \mathbf{B}_1 \dot{\hat{\Omega}}_1 \right) \mathbf{D}_1 \\
& -m_2 \left(\dot{\mathbf{B}}_2 \hat{\Omega}_2 + \mathbf{B}_2 \dot{\hat{\Omega}}_2 \right) \mathbf{D}_2 \\
& +\mathbf{f}_{ext_1} + \mathbf{f}_{ext_2}
\end{aligned}
$$

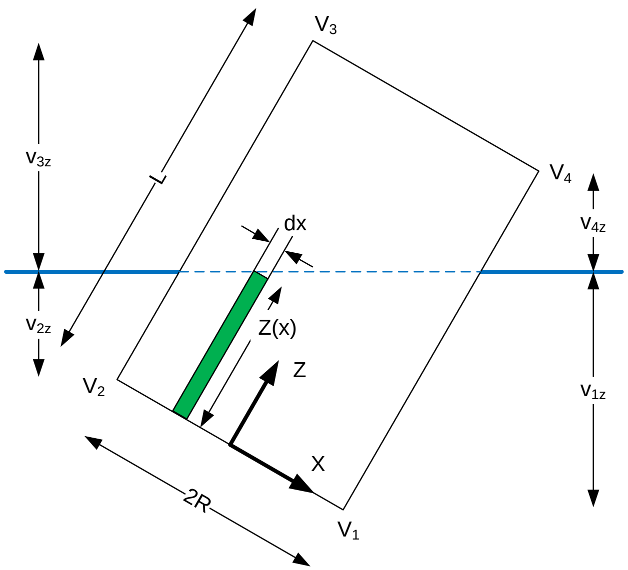
The translational acceleration of the joint (in navigation coordinates).

D_1 and D_2 are the distance between the body's CM and joint location. $\mathbf{D}_i = \mathbf{R}_{J_i} - \mathbf{R}_{cm_i}$



Buoyant / Gravity Forces & Moments

These forces and moments are easy to calculate with confidence for each cylinder.



$$Y(X) = 2\sqrt{R^2 - X^2}$$

$$V_{sub} = \sum_{k=1}^{n_x} dx\, Y(X)\, Z(X)$$

The buoyant effects are calculated in an **inclined coordinate system** and then transformed back to body coordinates.

The submerged volume of each cylinder is numerically estimated.

$$\mathbf{f}_{grav_i} = \begin{bmatrix} 0 \\ 0 \\ -m_i g \end{bmatrix}$$

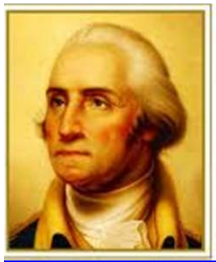
The gravity force is calculated in the global frame.

$$\mathbf{F}_{grav_i} = \mathbf{B}_i^T \mathbf{f}_{grav_i}$$

Then it is converted to the body frame for the moment calculation

$$\mathbf{M}_{grav_i} = (\mathbf{R}_{cmsys_i} - \mathbf{R}_{cm_i}) \times -\mathbf{F}_{grav_i}$$

The gravity moment is calculated in the body frame.



The Composite Pointing Error

A measure of the angular difference between the payload pointing vector and the aim point vector.

$$\tilde{\mathbf{p}}_z = \mathbf{B}_2 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

Unit vector of the payload's z axis in the navigation frame.

$$\tilde{\mathbf{r}}_{aim} = \begin{bmatrix} \cos r_z \sin r_v \\ \sin r_z \sin r_v \\ \cos r_v \end{bmatrix}$$

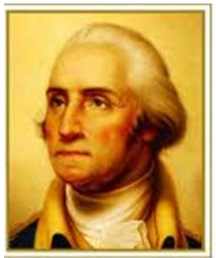
Unit vector of the commanded aim point in the navigation frame.

$$\boxed{\epsilon_{aim} = \operatorname{atan2}\left(\operatorname{abs}(\tilde{\mathbf{p}}_z \times \tilde{\mathbf{r}}_{aim})\,,\ (\tilde{\mathbf{p}}_z \cdot \tilde{\mathbf{r}}_{aim})\right)}$$

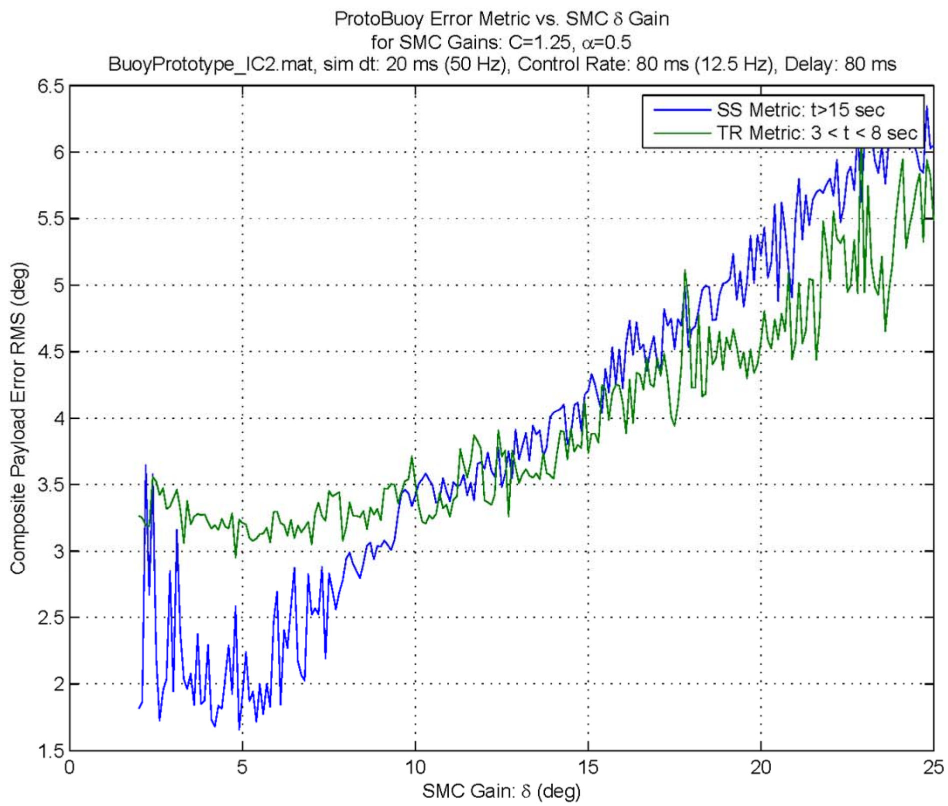
This is the primary metric of how well the buoy & control law system is performing.

An initial goal for desired system performance are:

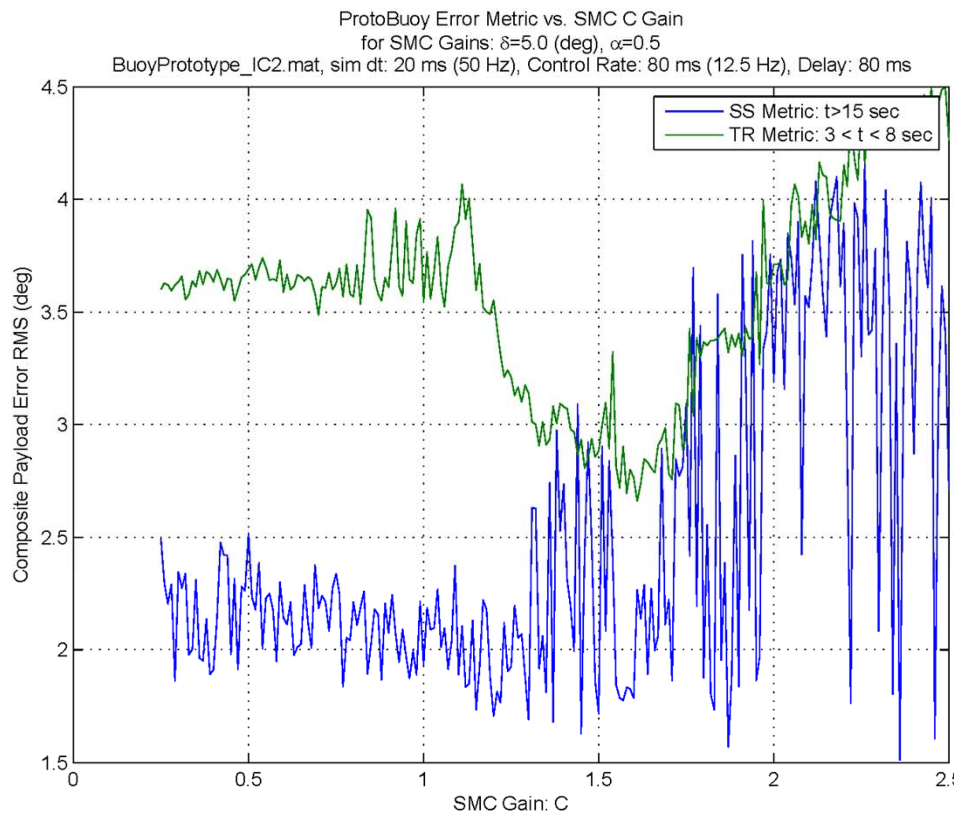
$$\begin{array}{lll} \epsilon_{aim} & < 10^\circ & \text{Threshold} \\ \epsilon_{aim} & < 5^\circ & \text{Goal} \end{array}$$



SMC Gain Search (δ and C)

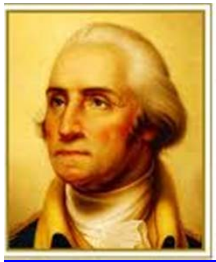


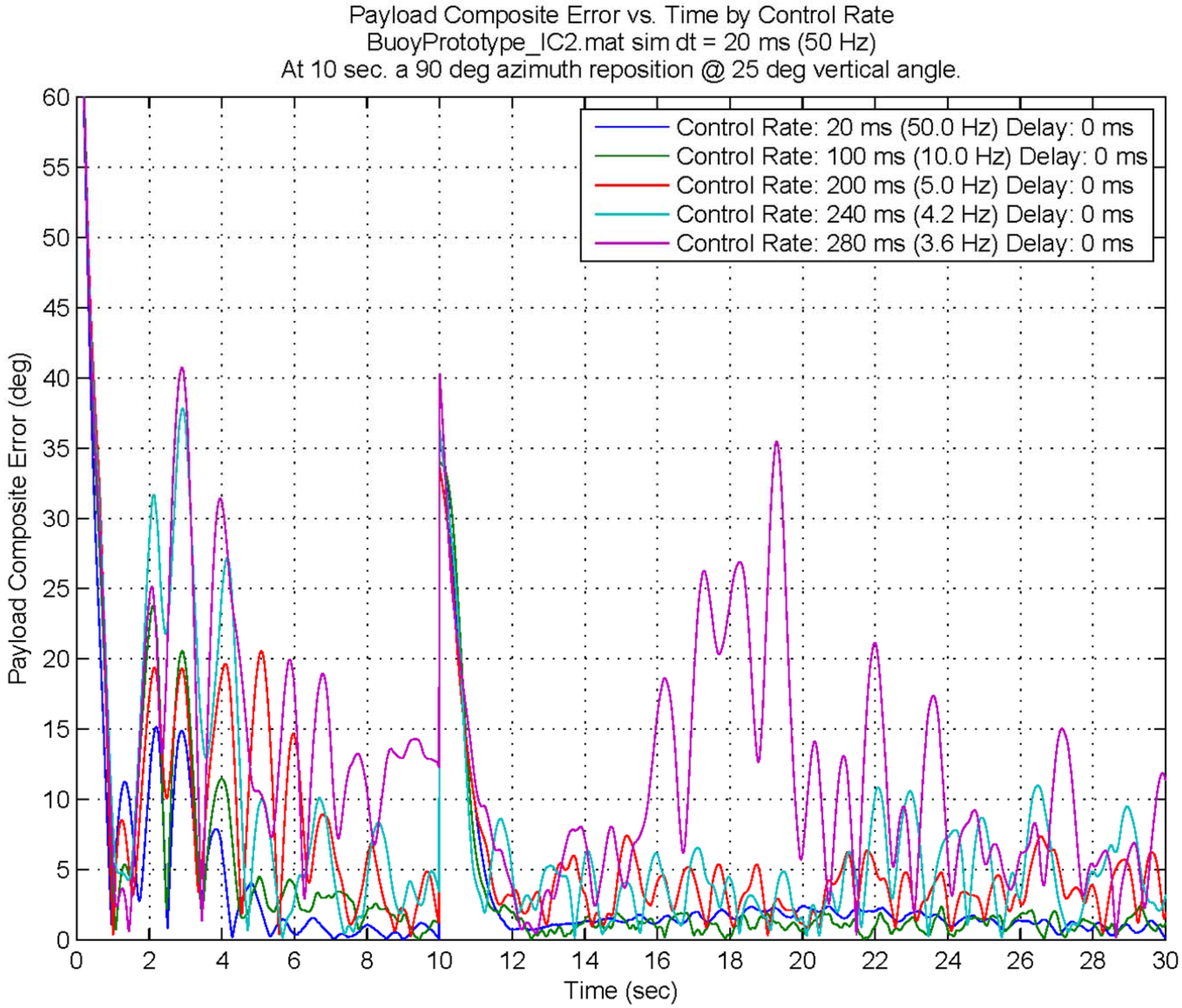
TR & SS Performance Metric vs. δ Gain



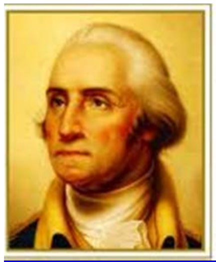
TR & SS Performance Metric vs. C Gain

Control Rate Effects

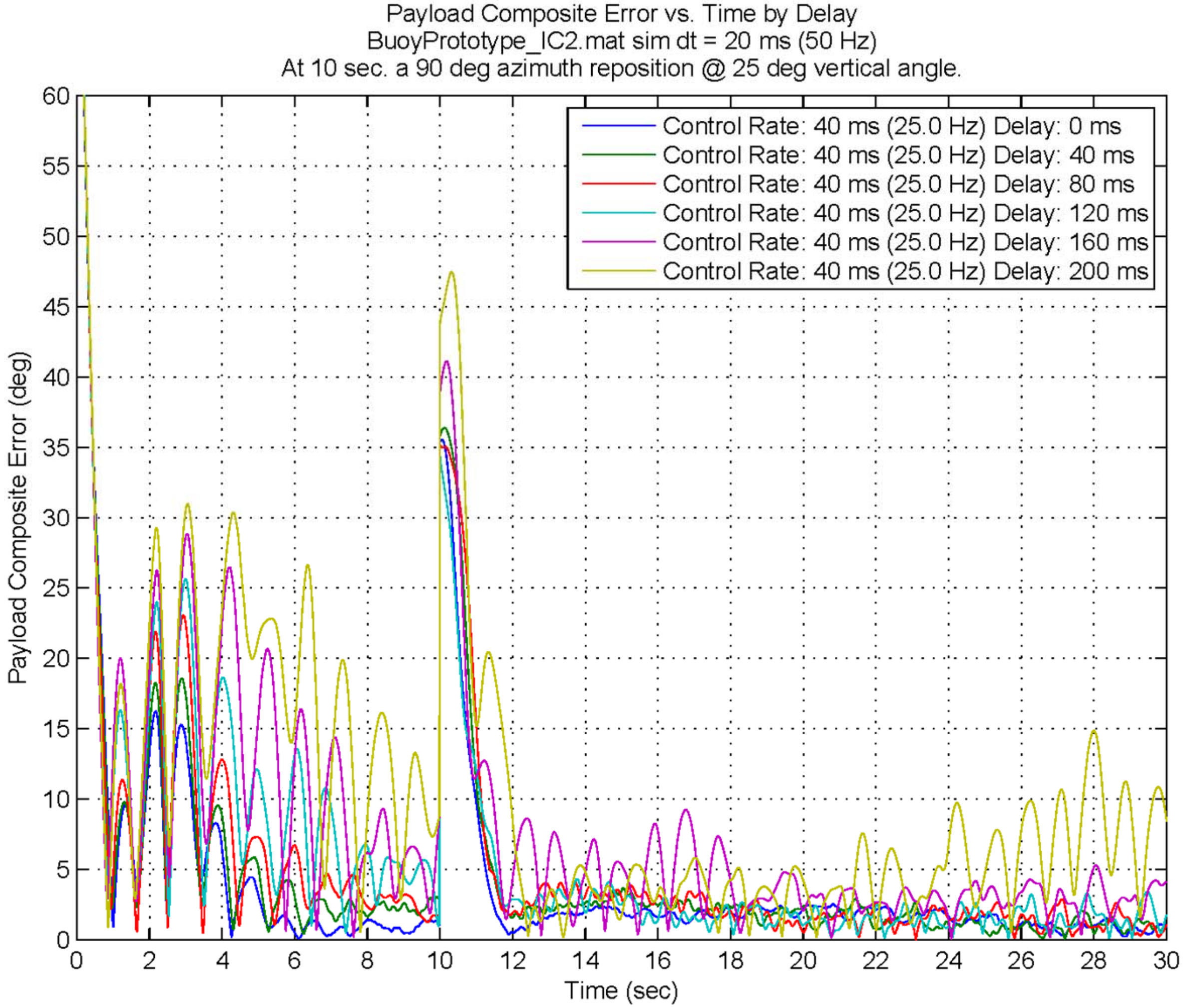




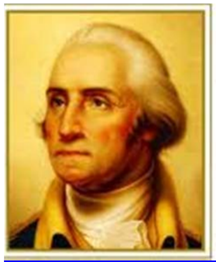
In simulation for the prototype buoy configuration, the control rate is not the limiting performance factor until it is longer than 100 milliseconds (10 Hz).



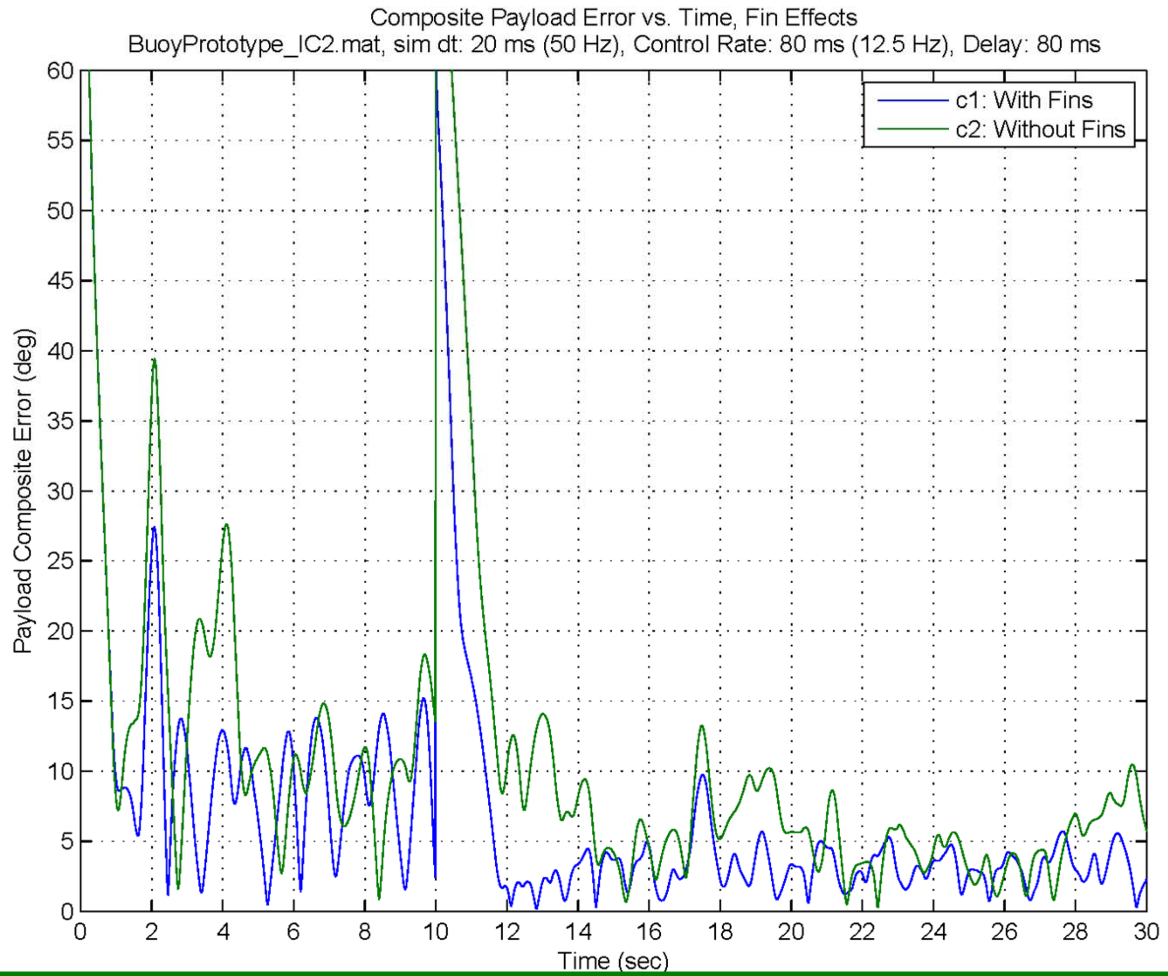
System Latency Effects



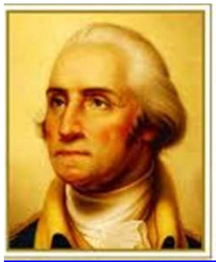
In simulation for the prototype buoy configuration, the system latency is not the limiting performance factor until it is longer than 120 milliseconds.



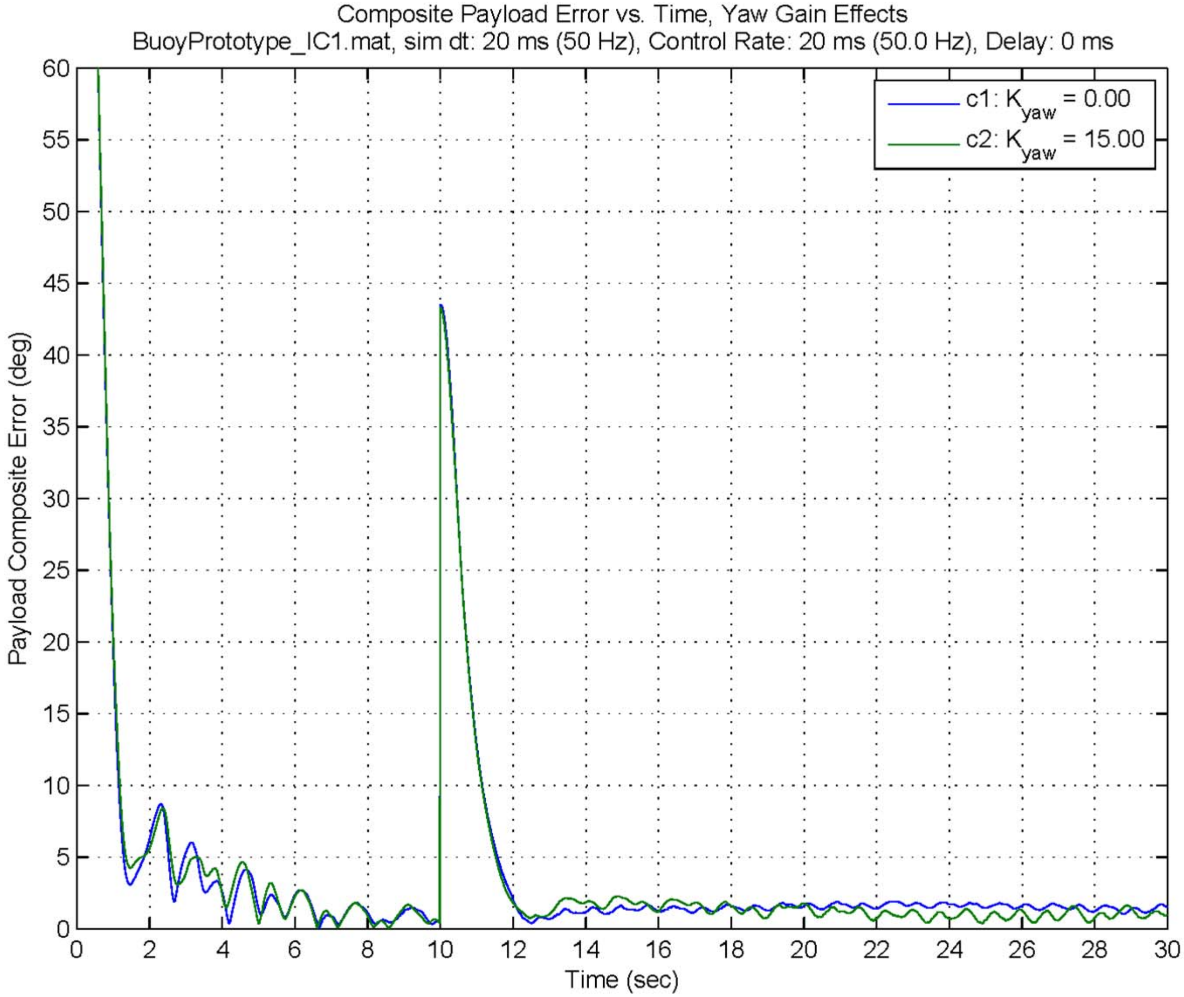
Fin Effects



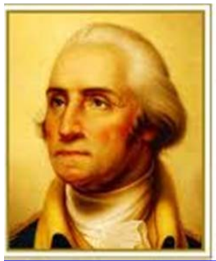
For the prototype buoy configuration, the fins did **not** significantly improve the system's performance in simulation. If added mass effects were included in the simulation, the fins might be shown to be more effective.



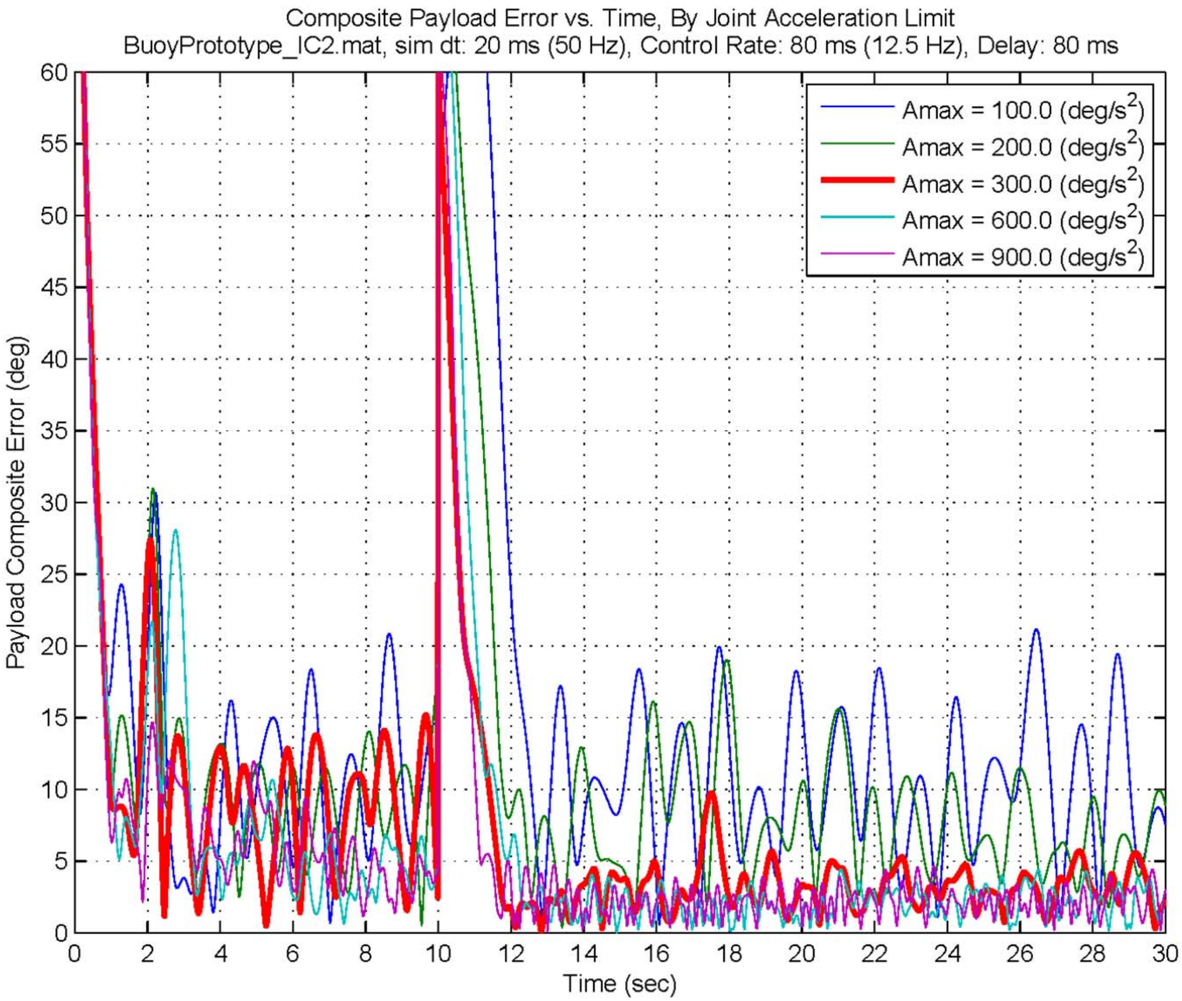
Yaw Damper Effects



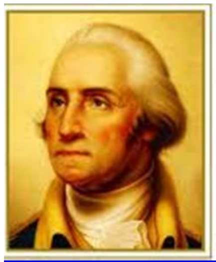
For the prototype buoy configuration the active yaw damper feature of the control law did not improve the system performance.



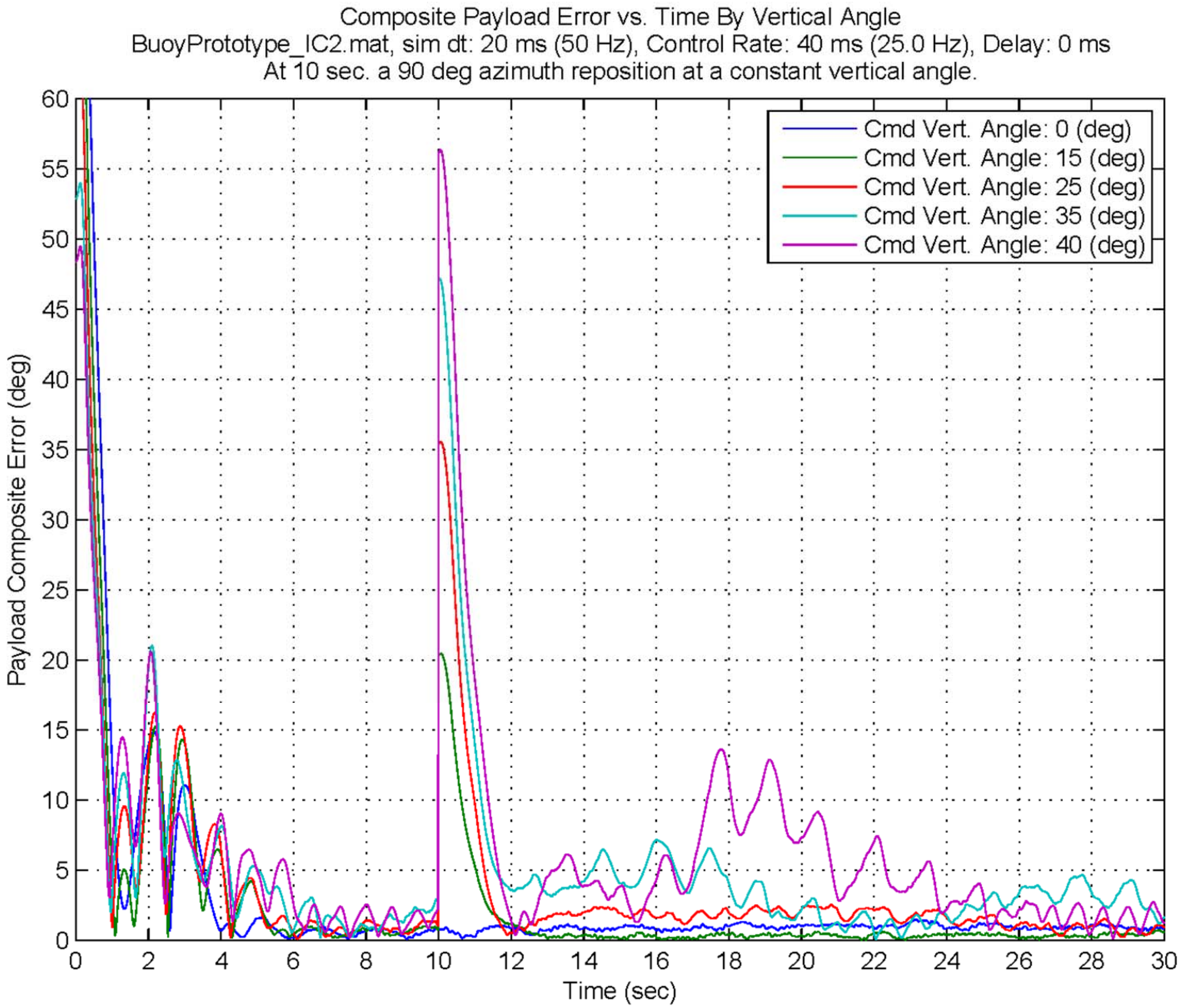
Acceleration Limit Effects



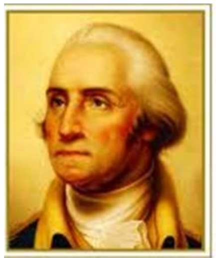
Increasing the axis acceleration limit to beyond 300 deg/sec^2, does not yield significant additional performance improvements.



Vertical Angle Effects

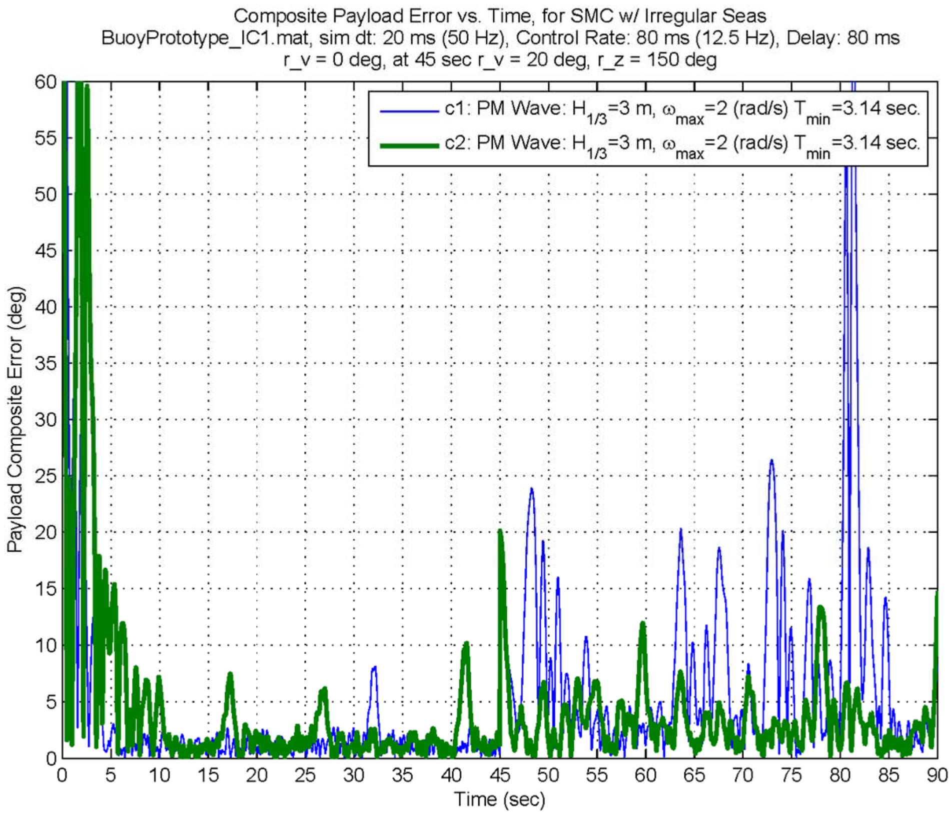


System performance gets noticeably worse for vertical angles greater than 35 degrees.

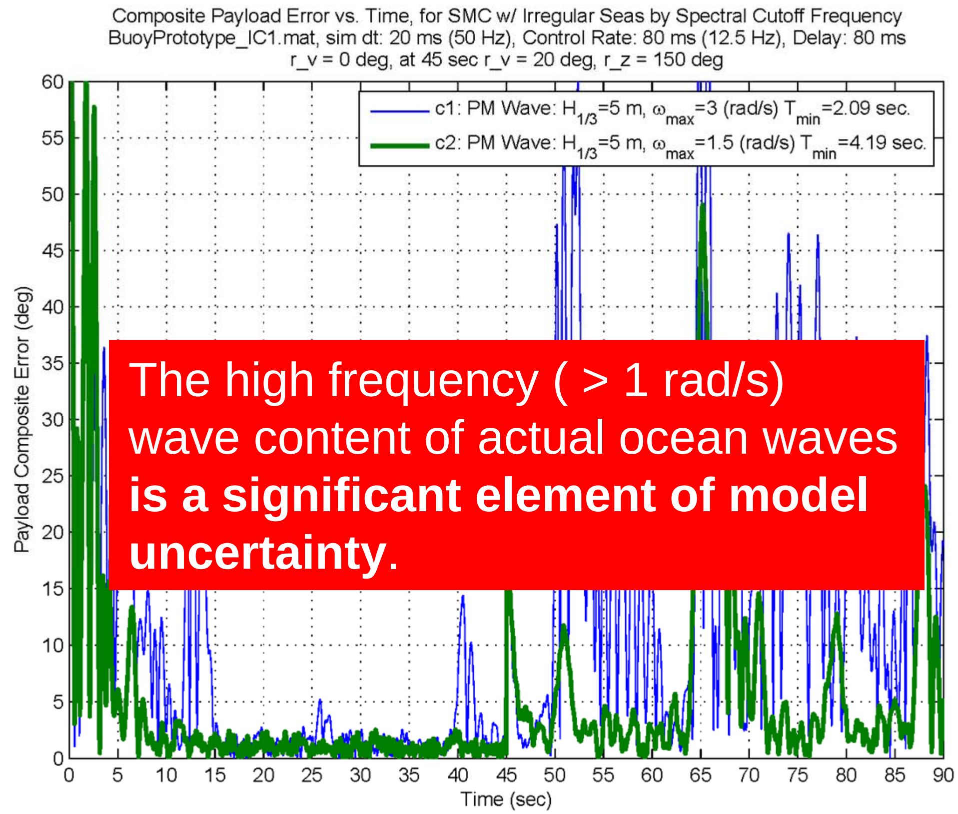


Other Irregular Sea Effects

Buoy Starts Horizontal and Payload Commanded to vertical for 45 seconds, then r_v = 20°.



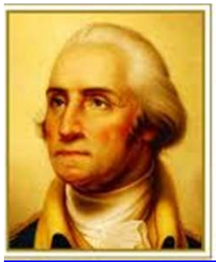
Two different irregular seas generated from the **same** PM spectrum.



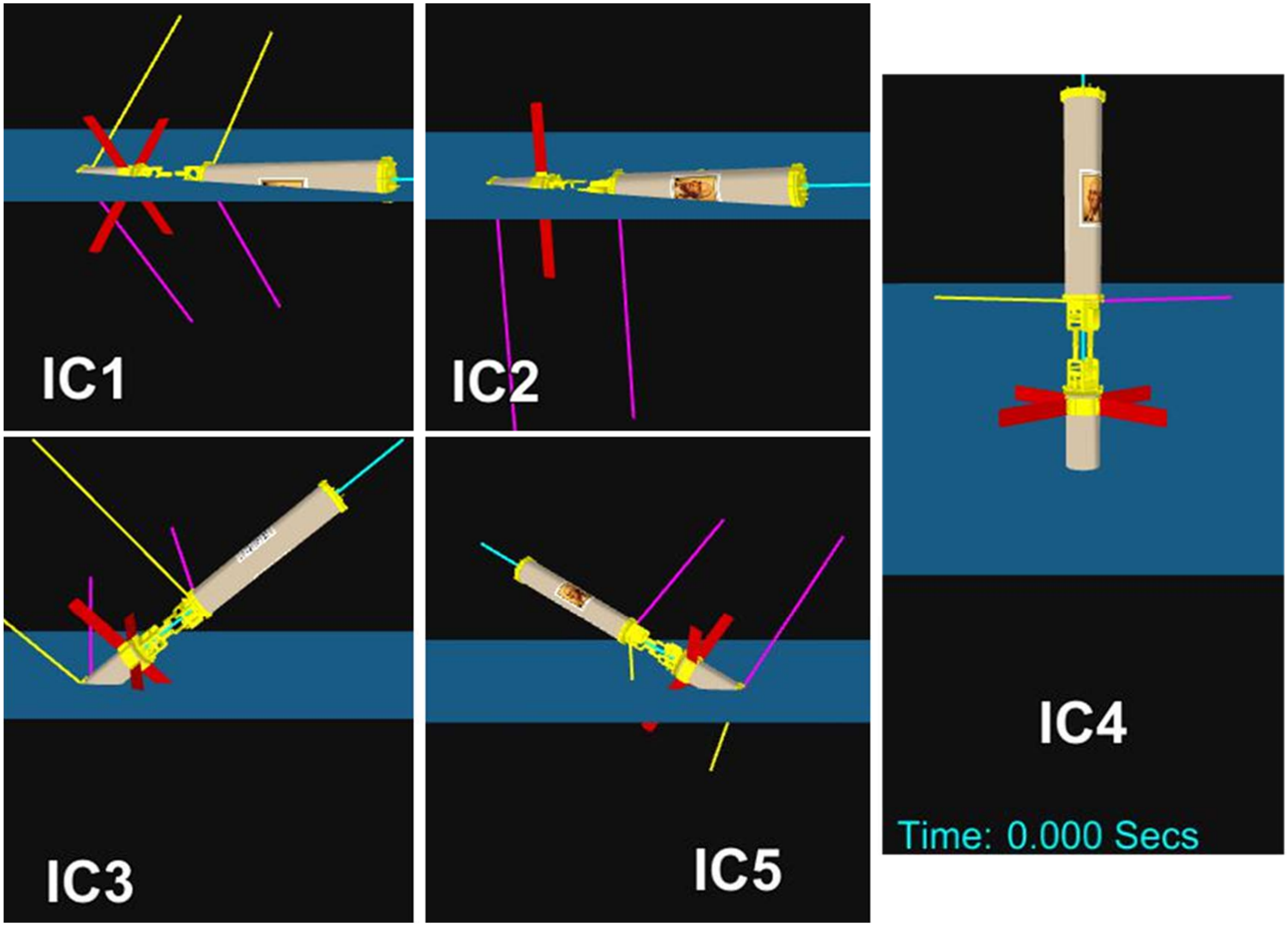
The high frequency (> 1 rad/s) wave content of actual ocean waves **is a significant element of model uncertainty**.

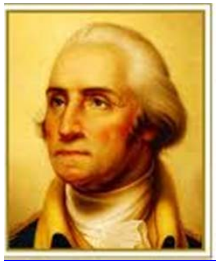
Same irregular seas wave form but with **different** spectral cutoff points.

A vertically stabilized payload is significantly more robust than a non vertical payload.
The high frequency content (> 1 rad/s) of the wave spectrum **is a critical factor** in system performance.

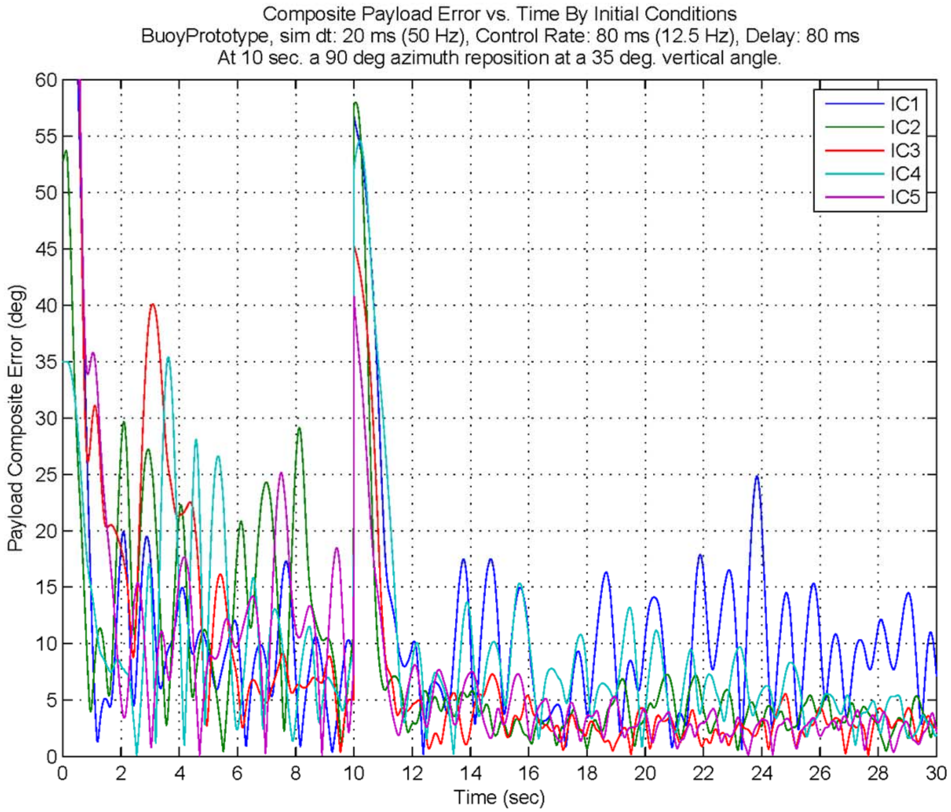


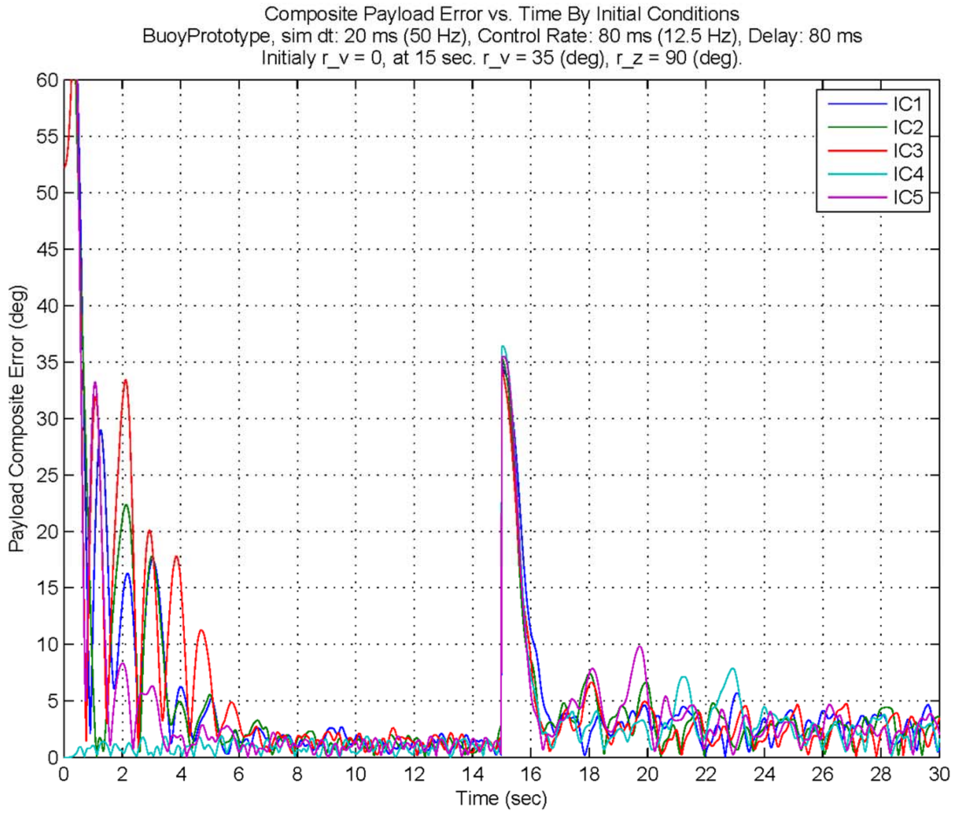
The Initial Conditions





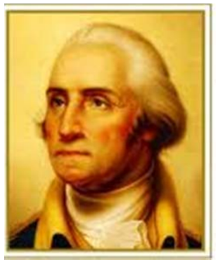
Initial Condition Effects



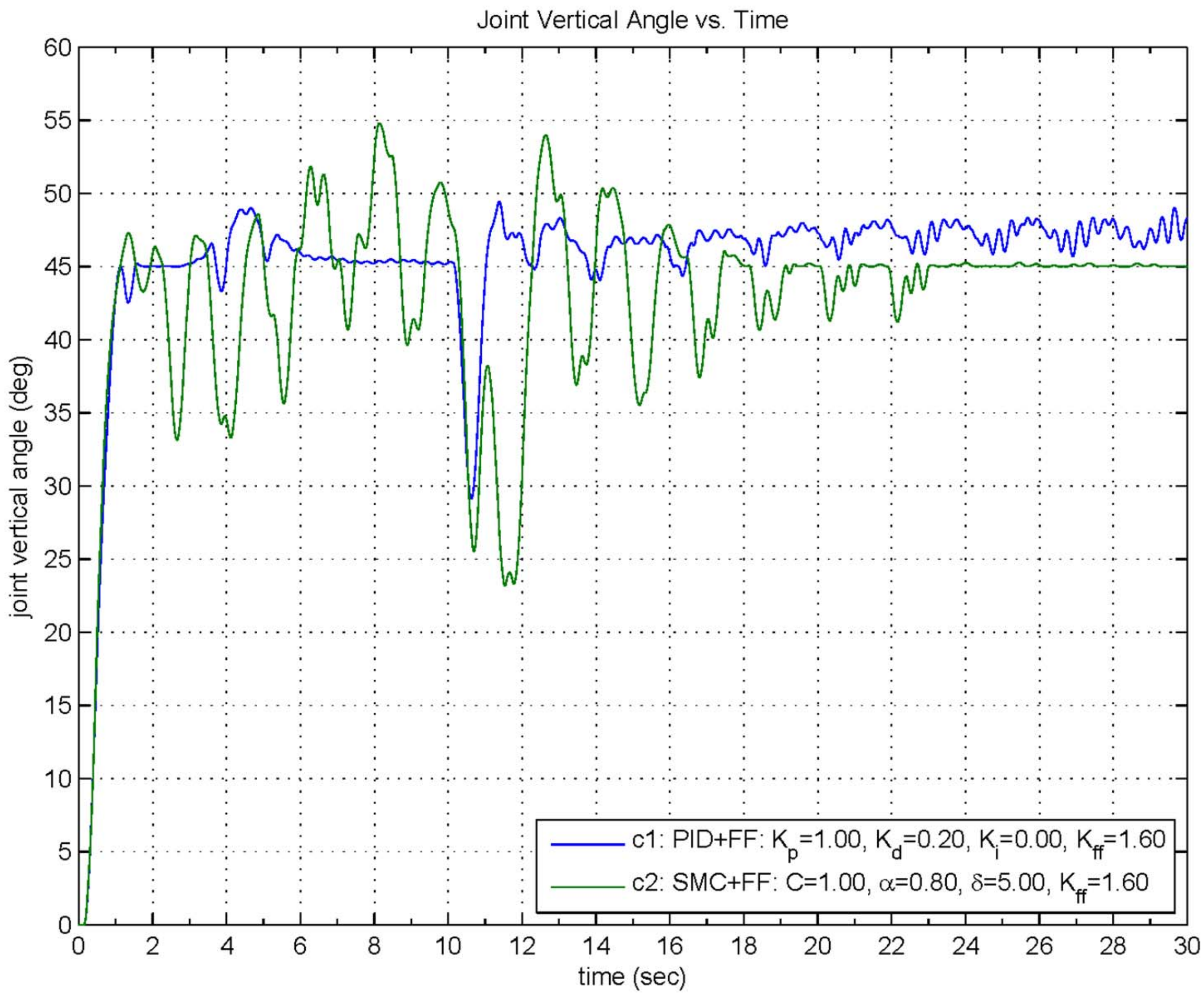


Initial conditions 1 and 2 are qualitatively very similar (buoy almost flat on the surface). One resulted in good performance (IC2) and other in poor performance (IC1). The reason for this is not understood.

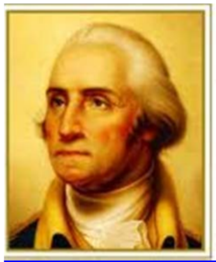
Stabilizing the system about the vertical before moving to a non-vertical angle, removes all the effects of the initial conditions.



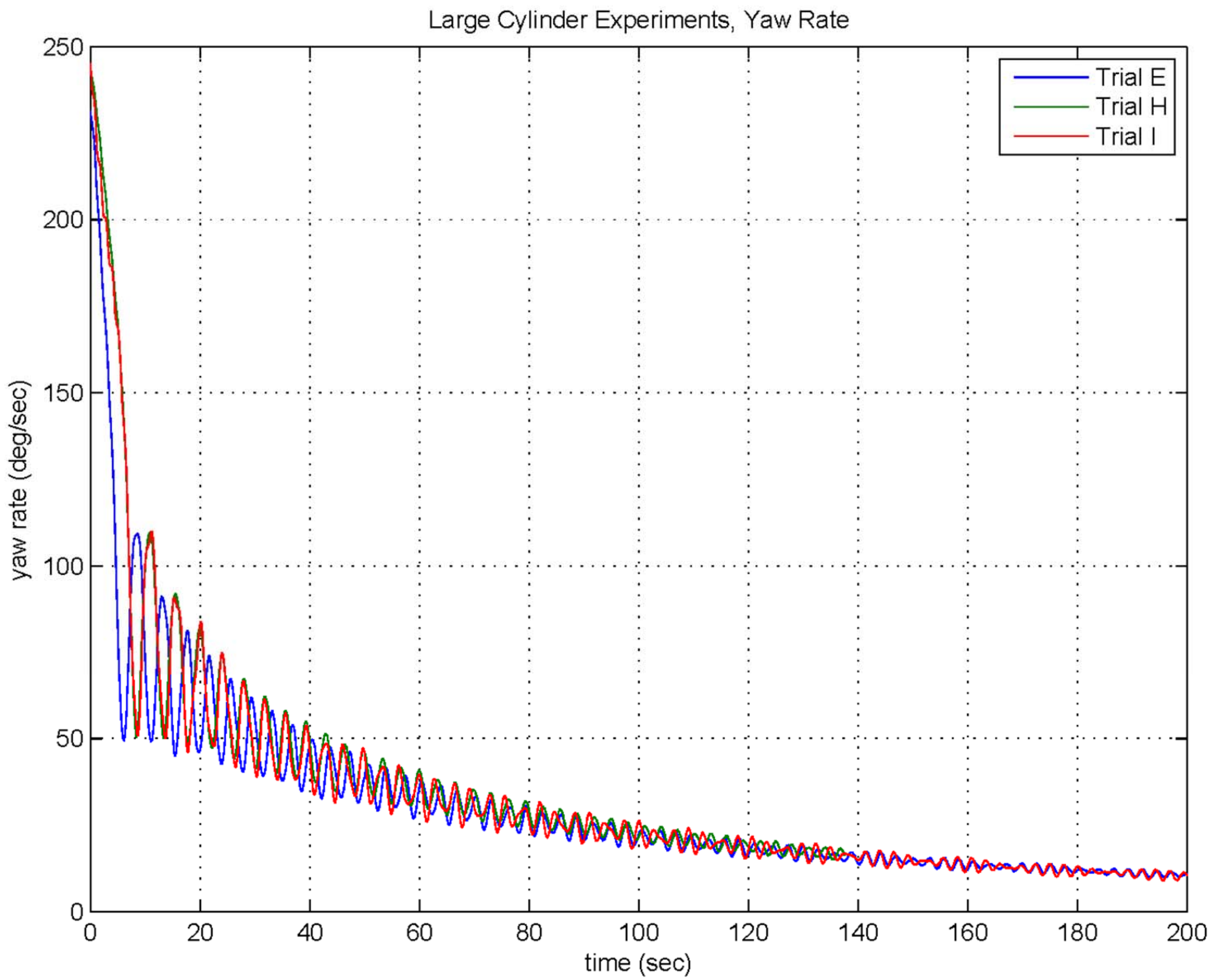
Joint Motion in SMC vs. PID Control

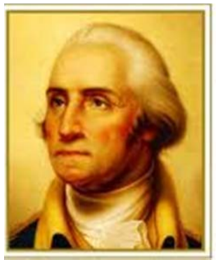


The SMC controller uses more joint motion to stabilize and control the buoy than the PID controller.



Large Cylinder Yaw Rate Trials



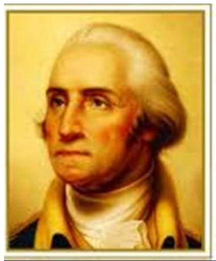


Model Simulation Loop

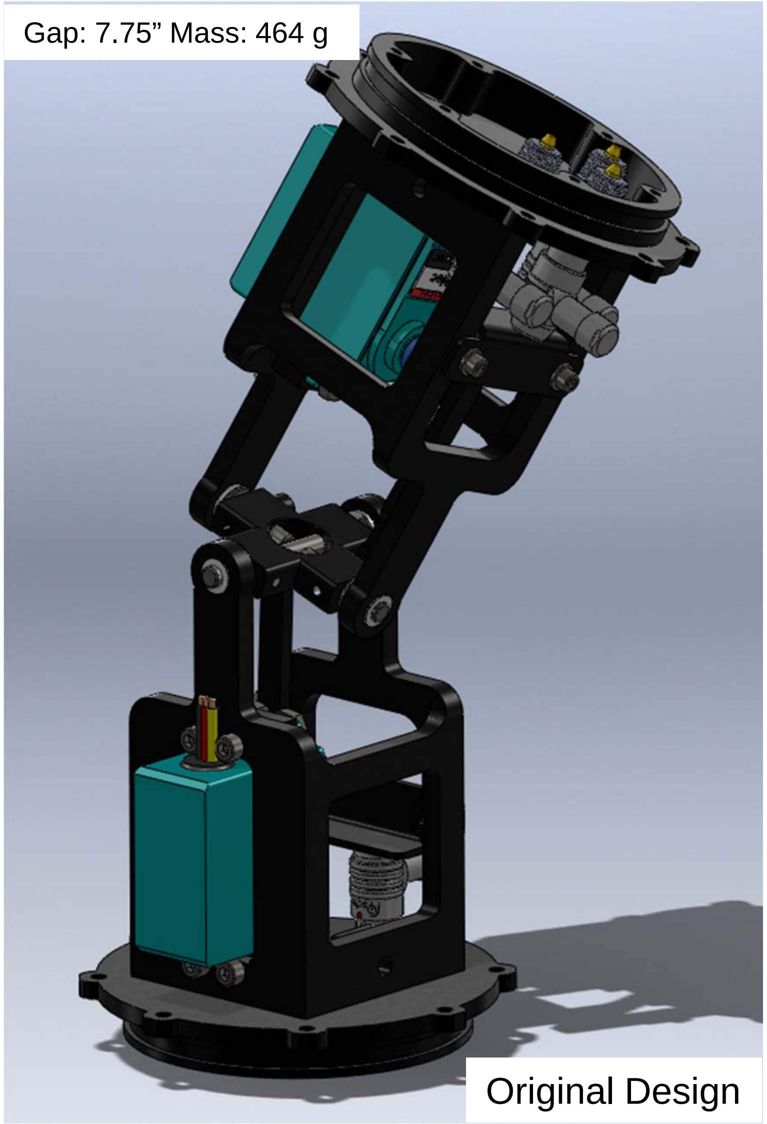
- Calculate new control signals*
- Update Joint Position, Velocity, & Acceleration.
- Calculate the Forces and Moments on the Bodies
- Calculate the Rotational Accelerations
- Calculate the Translational Accelerations
- Integrate the Velocity and Position States Forward

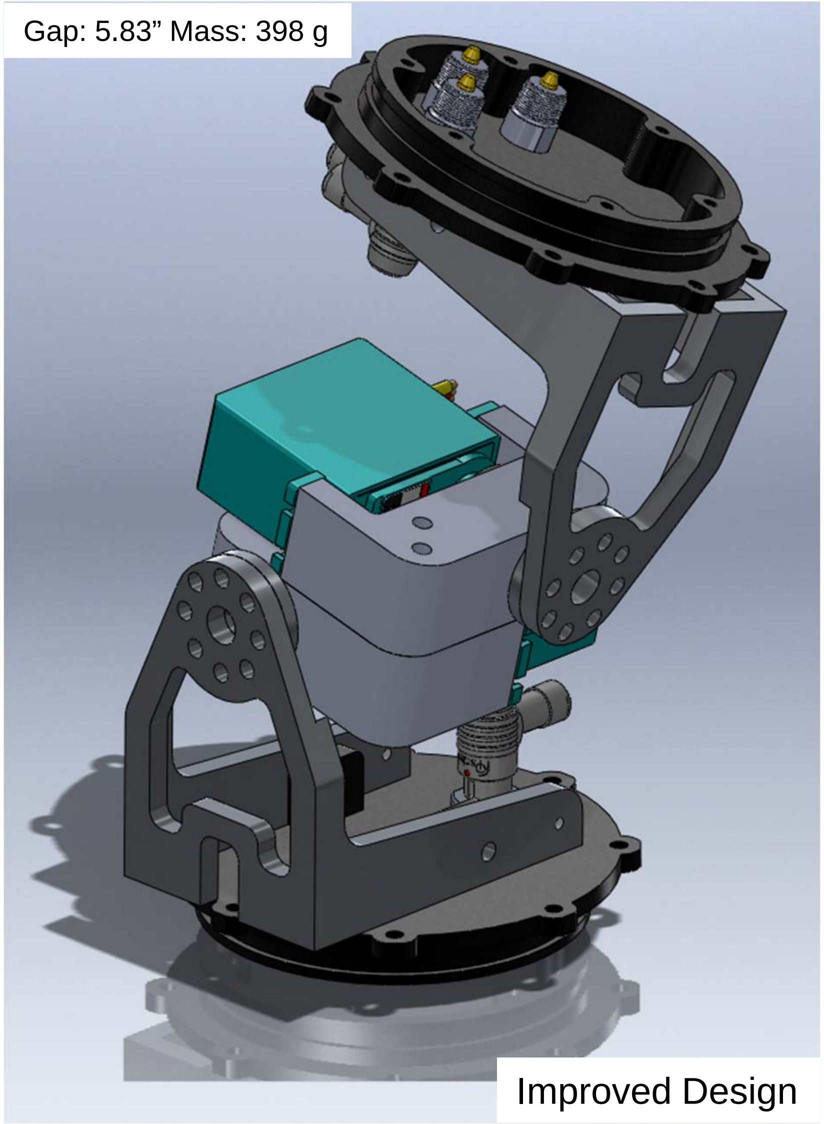
* Does not occur at every simulation step.

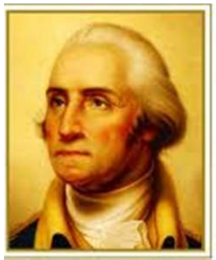
Large Wave Tank Images



An Improved Driven Universal Joint Design

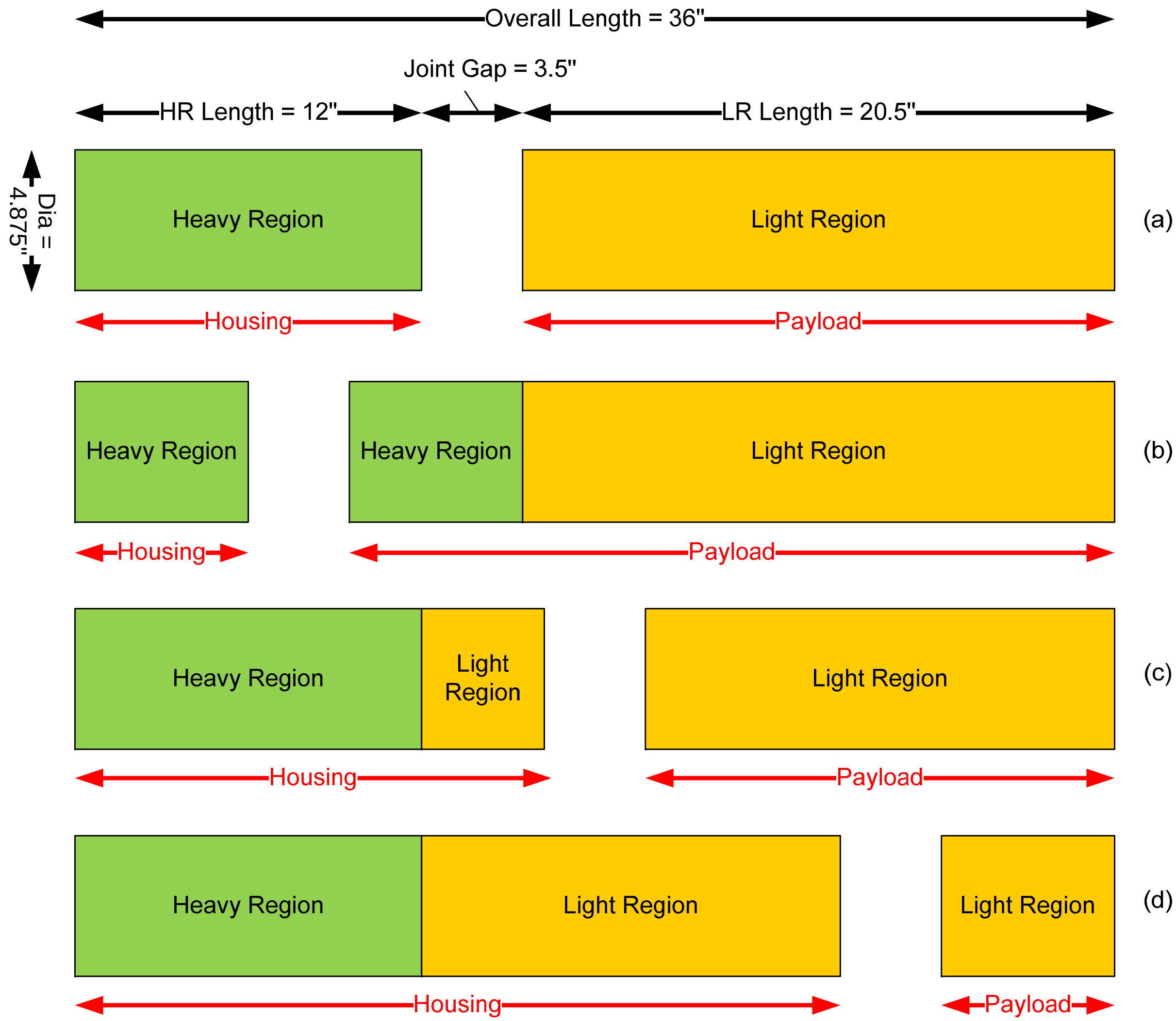


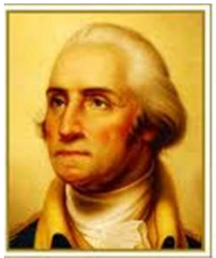




Other Buoy Models

Four buoys with the “same” mass distribution and different joint locations.





The Prototype Buoy Configuration