
ES360

Introduction to Controls Engineering

Lessons 1 to 11

Name: _____

Section: _____

Instructor: _____

Lesson	Topic	MATLAB Files Required
1a	Complex Number & Quadratic Equation Review	
1b	Block Diagram Reduction	
1c	Introduction to the Frequency Domain	wavgen.m
2a	Basic Element Types	
2b	First Order Transfer Functions	
3a	Input Types and the Final Value Theorem	
3b	Second Order Systems' Time Response	MATLAB Only
4	Second Order Time Response Calculations	
5a	Graphical Analysis of Second Order Responses	
5b	Measurement of a Second Order Response	
5c	Modeling the F/A-18 Landing Gear	f18.mdl & f18landing.m
6a	Introduction to Feedback	
6b	Introduction to Controllers	
7	Modeling and Feedback of a Gun Turret	MATLAB & SIMULINK
8	Disturbances and Actuator Limitations	gun_barrel_elev.mdl
9	PID Control of a Generator Set	generator.mdl
10	PID Control and Discrete Sampling	cruise_missile.mdl
11	Hardware Implementation of PID Speed Control	

United States Naval Academy
Weapons and Systems Engineering Department

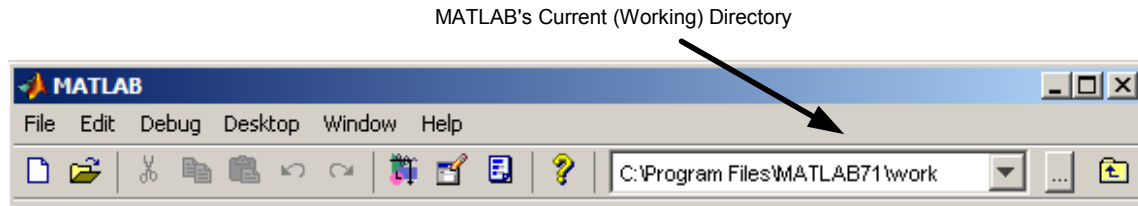
LT Roger Cortesi, USN

Revision 2.2
December 2006

Classroom	Door Combination

Setting MATLAB's Current Directory (a.k.a. the Working Directory)

Set MATLAB's current directory to a folder which you can save files in. In MATLAB's main window the current directory is displayed in the upper right corner. Use the "..." button to change it.



Entering a transfer function in MATLAB

There are two methods of entering a transfer function in MATLAB. For example, the transfer

function $G(s) = \frac{25}{s^2 + 5s + 25}$ could be entered by either:

MATLAB Commands

Remarks

Method 1:

```
>> s=tf('s');
>> G = 25/(s^2+5*s+25)
Transfer function:
      25
-----
s^2 + 5 s + 25
```

The first line specifies 's' as the transfer function's variable. This command only needed once per MATLAB session.

The second line specifies the transfer function G as a function of s. If you don't terminate the line with a semicolon, MATLAB will display the transfer function. This makes it easy to confirm that the transfer function was entered correctly.

Method 2:

```
>> G = tf([25],[1 5 25])
Transfer function:
      25
-----
s^2 + 5 s + 25


>> G = tf([25 0],[1 0 25])
Transfer function:
      25 s
-----
s^2 + 25
```

Use the MATLAB function "tf". It takes two arguments, a list of coefficients for the numerator and list for the denominator. Again, omitting the semicolon allows you to verify that the transfer function correctly was entered correctly.

The second example demonstrates that the coefficient of 0 must be included to skip terms in numerator and denominator.


Starting SIMULINK

SIMULINK can be started by:


- 1) Opening a SIMULINK model file (model files use the .mdl extension).
- 2) Starting MATLAB and clicking on the  icon in the tool bar.

The SIMULINK Library Browser

SIMULINK models are made up of different elements connected in a block diagram. The SIMULINK Library Browser is catalog of all the elements available to the model.

View the library browser with the  icon or by selecting the “Library Browser” menu option in the “View” menu of a model file.

Creating a New SIMULINK Model

Open the SIMULINK Library Browser. Click on the “New Model” icon  or select “New... → Model File...” from the Library browser’s “File” menu. This will open a blank model window.

Setting the Start and Stop Time of a Simulation

The start and stop time of a simulation is set in the model file’s “Configuration Parameters” window.

(“Simulation” menu → “Configuration Parameters”, or press Ctrl-E)


SIMULINK Printing Problems

Sometime when printing a plot generated by SIMULINK, the line will be printed in a very light shade of grey. This is makes it very difficult to read the plot. This happens when MATLAB sends color data to a black and white printer. The black and white printer prints the yellow line as a very light shade of grey.


To fix this, go to the “**general**” setting in the preferences window (“File” menu → “Preferences...”). Set “Figure Window Printing” to “**Always send as black and white**”.

Make sure you use the “File” menu from the SIMULINK model file’s window.

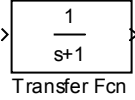








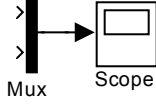

SIMULINK Scope Block NOT Plotting All the Results

If a scope block does not seem to be displaying all the results from the simulation, try clicking on the autoscale button  at the top of the plot to resize the plot’s axes.



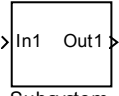


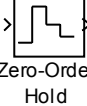
In some simulations the scope block will not plot data at the beginning of the simulation. For example, in a 20 second simulation, it might only plot the last 14 seconds of data. By default a scope block will only display the last 5,000 data points. If the simulation used more that 5,000 data points, then the earlier points will not be plotted.

To change this setting, open the ‘Scope’ Parameters window by clicking on the  icon in the top right corner of the scope’s plot. Select the “Data History” tab and uncheck the “Limit data points to last: x” checkbox. You need to make this change to each of the affected scopes.

Useful SIMULINK Blocks

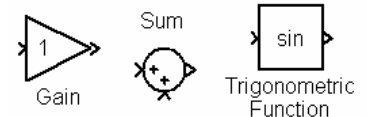
Block	Location in the Library and Notes
	<p>Location: Continuous</p> <p>Double click on the block to set the transfer function. The numerator and denominator are set in bracket format. For example, [1 2 3] = $s^2 + 2s + 3$. See method 2 for specifying a transfer function.</p> <p>It may be necessary to make the block bigger to display its current transfer function.</p>
	<p>Location: Commonly Used Blocks & Math Operations</p> <p>Double click on the block to set the gain.</p> <p>If the block is too small to display its current value, it displays “-K-“. Make the block bigger to display the current gain value.</p>
	<p>Location: Commonly Used Blocks & Math Operations</p> <p>Double click on the block to set the number of inputs and their signs. The string “ ++” produces the format displayed here. “+++” has three positive inputs. “ +-“ is a typical junction for negative feedback loops. The “ ” character means “no input at this location”. Try other combinations of the characters “ ”, “+”, and “-“.</p>
	<p>Location: Sources</p> <p>This block generates a step function. Double click on the block to set its initial value, final value, and step time.</p> <p>By default, the step occurs at time=1. Double click on the step block to change the step time to t=0. You must make this change every time you use a new step function.</p>
	<p>Location: Commonly Used Blocks & Sinks</p> <p>This block generates a plot of its input for the duration of the simulation.</p> <p>Double click on the block to display the plot.</p> <p>The  icon auto scales the plot. The  and  icons allow zooming in on the x and y dimensions of the plot.</p>
	<p>Location: Signal Routing</p> <p>This block combines two or more signals into a single signal path. The signals are unchanged by this combination. It is useful for plotting multiple signals on the same scope.</p> <p>Double click the block to set the number of input signals.</p> 
	<p>Location: Continuous</p> <p>This block takes the derivative of its input.</p>

MATLAB and SIMULINK Help

 <p>Integrator</p>	<p>Location: Commonly Used Functions & Continuous</p> <p>This block takes the integral of its input. The default value for its initial condition is zero.</p> <p>Double clicking on the block allows the initial value to be set to some other value or driven from an external source.</p>
 <p>Display</p>	<p>Location: Sinks</p> <p>This block displays the value of its input. It is useful for displaying a final value at the end of the simulation.</p>
 <p>Subsystem</p>	<p>Location: Commonly Used Blocks & Ports/Subsystems</p> <p>This block allows another model to be completely embedded in the model. Double clicking on the block opens another model window to edit the subsystem.</p> <p>This block is useful for keeping the main model file from becoming too cluttered. Adding input and output blocks to the subsystem will add more ports to the subsystem.</p>
 <p>Sine Wave</p>	<p>Location: Sources</p> <p>This block generates a sine wave. Double clicking on the block allows the amplitude and frequency to be set.</p>
 <p>Saturation</p>	<p>Location: Commonly Used Blocks & Discontinuous</p> <p>This block limits the signal to be between an upper and lower value (inclusive). Double click on the block to set the upper and lower limits.</p>
 <p>Zero-Order Hold</p>	<p>Location: Discrete</p> <p>This block periodically samples the input. Its output for the sample period is the value of previously sampled input.</p> <p>Double click on the block to set the sampling period.</p>

Performing Basic Math Operations

The *Library Browser* branch *Math* contains commonly used math functions. To multiply a signal by a constant, use the *gain* block. To change the gain, double click on the gain block and change the *Gain* field of the *Block Parameters* window. To add two or more signals, use a *Sum* block. By double clicking on the *sum* block you can change the shape of the block, the number of inputs, and sign of each input. Simulink also contains nonlinear functions including trigonometric functions. By double clicking, you can select any common trigonometric function. NOTE: Simulink and MATLAB always assume RADIANS as the units of all angles for trigonometric functions!



Basic MATLAB Workspace Commands

who	lists the names of all variables in the workspace
whos	lists all variables in the workspace along with their size and data type
clear	removes all variables from workspace
clear <i>name</i>	removes the variable name
save <i>name</i>	saves all variables in the workspace in the disk file name
load <i>name</i>	loads variables from disk file name into the workspace
dir	lists the files in the disk current directory

Basic MATLAB Plotting Commands

plot(x,y)	plots data in vector y versus the data in vector x
plot(x,y,'s')	plots data in vector y versus x with line attributes 's'
plot(x1,y1,'s1',x2,y2,'s2')	plots y1 versus x1 with line attributes 's1' and y2 versus x2 with line attributes 's2'

The following table summarizes basic line attributes. An attribute string can contain one character from each column:

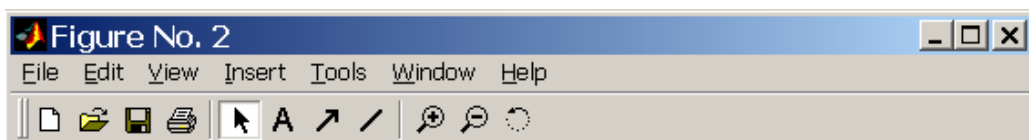
Basic MATLAB Line Attributes		
Color	Marker Style	Line Style
b blue	. point	- solid
g green	o circle	: dotted
r red	x x-mark	-. dashdot
c cyan	+ plus	-- dashed
m magenta	* star	
y yellow	s square	
k black	d diamond	

You can create and manipulate multiple figure windows using the following commands:

figure(n)	switch to the nth figure window, making it the active figure, create figure window n if necessary
hold on	hold active figure so that plot commands are cumulative
hold off	release active figure hold so that the next plot command erases existing data
clf	clear the active figure window

The active figure can be labeled and annotated using the following commands:

xlabel('string')	label the x-axis
ylabel('string')	label the y-axis
title('string')	title the plot
grid	add grid lines to the plot
text(x,y,'string')	place text inside the plot at point (x,y)
gtext('string')	place text by clicking the mouse at on the plot
legend('string1','string2',...)	annotate plot with 'string' descriptions of multiple curves



To enable interactive plot editing, click arrow pointing toward the upper left. In the above figure, the white box around this icon indicates that interactive plot editing has been selected.

Useful MATLAB Operators and Functions

Operators		Basic Functions		Trig. Functions	
$\mathbf{x+y}$	Addition	round(x)	Convert to Integer	sin(x)	Sine of x
$\mathbf{x-y}$	Subtraction	sqrt(x)	Square root of x	cos(x)	Cosine of x
$\mathbf{x*y}$	Multiplication	log(x)	Natural logarithm of x	asin(x)	Arcsine of x
$\mathbf{x/y}$	Division	log10(x)	Logarithm base 10 of x	acos(x)	Arccosine of x
$\mathbf{x^y}$	Power	exp(x)	Exponential of x	tan(x)	Tangent of x
Matrix Specific		abs(x)	Absolute value of x	atan(x)	Arctangent of x
$\mathbf{x'}$	Transpose	imag(x)	Imaginary part of x	atan2(x,y)	Four quadrant arctangent of (x/y)
inv(x)	Inverse	real(x)	Real part of x		
det(x)	Determinant	conj(x)	Complex conjugate of x	angle(x)	Angle of x

Control Specific Commands

Creating and analyzing linear models

`sys = tf(num,den)` Create transfer function from numerator and denominator polynomials
`sys = zpk(z,p,k)` Create transfer function from with zeros z, poles p, and gain k
`[z,p,k] = tf2zpk(num,den)` Find the zeros z, poles p, and gain k from num and den polynomials

Manipulating Polynomials

`[r,p,k] = residue(num,den)` Partial fraction expansion in terms of residues r, poles p, and direct term k
`x = roots(y)` Create vector x containing roots of polynomial with coefficients defined in vector y
`y = poly(x)` Create vector y of polynomial coefficients with roots defined in vector x

Model dynamics

`dcgain(sys)` Calculate D.C. (low frequency) gain
`bandwidth(sys)` Calculate system bandwidth
`pole(sys)` Calculate system poles
`zero(sys)` Calculate system poles (transmission) zeros
`damp(sys)` Calculate natural frequency and damping of system poles

Time-domain analysis

`ltiview(sys)` Response analysis Graphical User Interface (LTI Viewer)
`step(sys)` Plot or calculate step response
`impulse(sys)` Plot or calculate impulse response

Frequency-domain analysis

`bode(sys)` Bode diagrams of the frequency response

Classical design

`sisotool(sys)` Single Input Single Output (SISO) design Graphical User Interface
`rlocus(sys)` Plot root locus
`rltool(sys)` Root locus interactive design tool

Complex Numbers

Recall that complex numbers have a real and an imaginary part. The imaginary part is usually written as a multiple of i . Where i is the square root of -1 . Many texts will use the letter j , instead of i , for the square root of -1 .

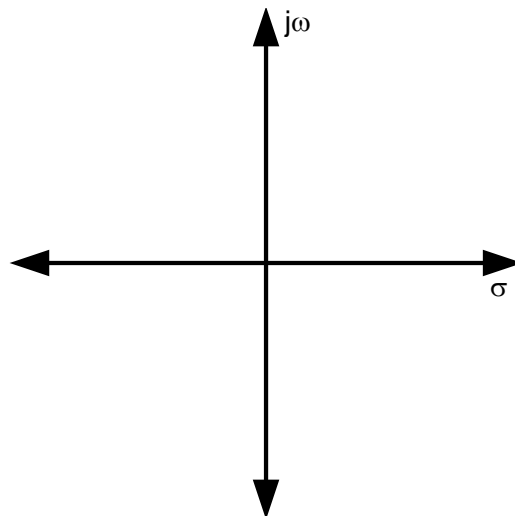
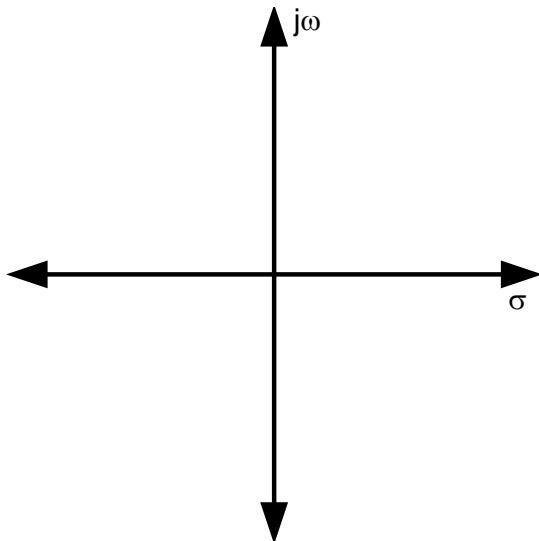
Complex numbers can be written in either rectangular form, $s = a + bi$ where the real and imaginary components are separate. Or they can be written in polar form as an angle and a magnitude, $s = c \angle d^\circ$.

Addition and subtraction of complex numbers by hand is easier in rectangular form.

Multiplication and division of complex numbers by hand is easier in polar form.

1) For each question find the numerical answer in rectangular and polar form. Plot the answers on the complex plane. **Calculate at least one part (the rectangular or polar answer) without a calculator.** The questions have been selected to make this easy.

		Rectangular	Polar	
a	$\sqrt{-1}$			
b	$\sqrt{-4}$			
c	$2 + 4i + 5 - 2i$			
d	$\sqrt{-2}$			
e	$(2 + 5j) \cdot (3 + 0.5j)$			
f	j^2			
g	i^3			
h	$(1 \angle 180^\circ) \cdot 2$			
i	$2 \angle 90^\circ + 3 \angle 270^\circ$			
j	$1 \angle 90^\circ \cdot 2 \angle 45^\circ$			



Quadratic Equations

Recall that quadratic equations have the form

$$a \cdot x^2 + b \cdot x + c = 0.$$

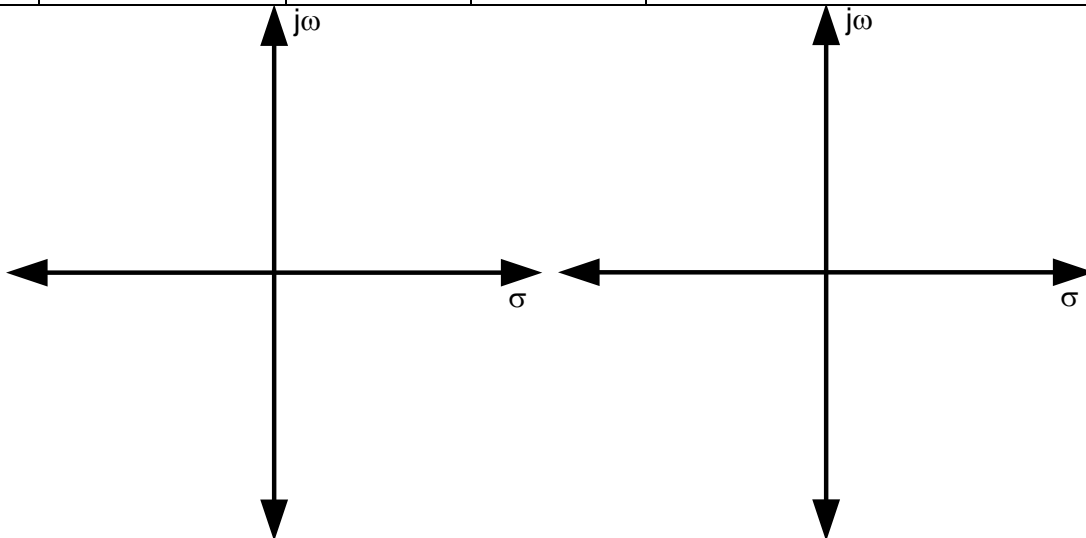
The two solutions to the quadratic equation are given by:

$$x = \frac{-b \pm \sqrt{b^2 - 4 \cdot a \cdot c}}{2 \cdot a}.$$

If $4 \cdot a \cdot c$ is greater than b^2 , then the solutions will have an imaginary component because the term under the radical will be negative.

2) For the quadratic equations below find both solutions and plot them on the complex plane. **Solving for both complex solutions of the quadratic equation is an excellent use of your calculator.**

		x_1	x_2	
a	$x^2 + 6x + 5 = 0$			
b	$x^2 - 6x + 5 = 0$			
c	$x^2 - 6x - 5 = 0$			
d	$x^2 + 2x + 5 = 0$			
e	$x^2 + 4 = 0$			
f	$x^2 - 4x + 4 = 0$			
g	$x^2 + 6x = 0$			
h	$2x^2 + 6x + 5 = 0$			
i	$x^2 + 5x + 7 = 0$			
j	$6x^2 + x + 7 = 0$			



Block Diagrams

Introduction and Multiplication

When building a mathematical model of a system it is useful to draw a diagram showing the relationship between the different components. This is typically done in the form of a block diagram. Block diagrams help break the problem down into manageable components.

Consider a simple spring. Hook's Law for a spring is

$$f = k \cdot x.$$

Given a force applied to the spring one can find the spring's displacement; or given an applied displacement one can find the resulting force. Figure 1 displays this relationship as a block diagram.

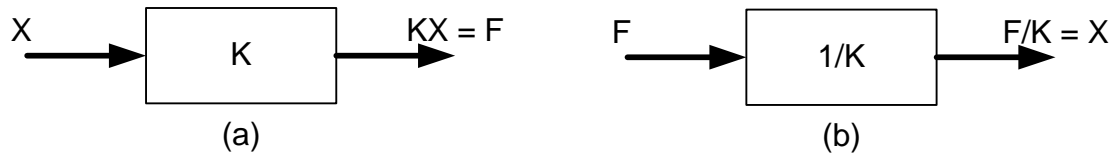


Figure 1: The block diagram for a spring showing displacement as the input and force as the output (a) and force as the input and displacement as the output (b).

The spring example illustrates that the **fundamental operation of the block diagram is multiplication**. The output of the block is the product of the input and the block itself. Figure 2 shows a pair of blocks connected and the resulting output.

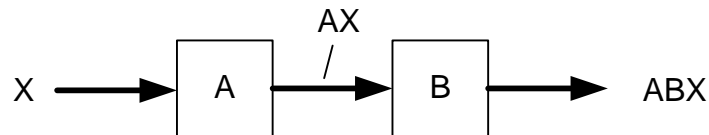


Figure 2: The output is the product of the input and the block.

Addition and Subtraction

Addition (and subtraction) is accomplished by a summing junction on the outputs of the blocks.

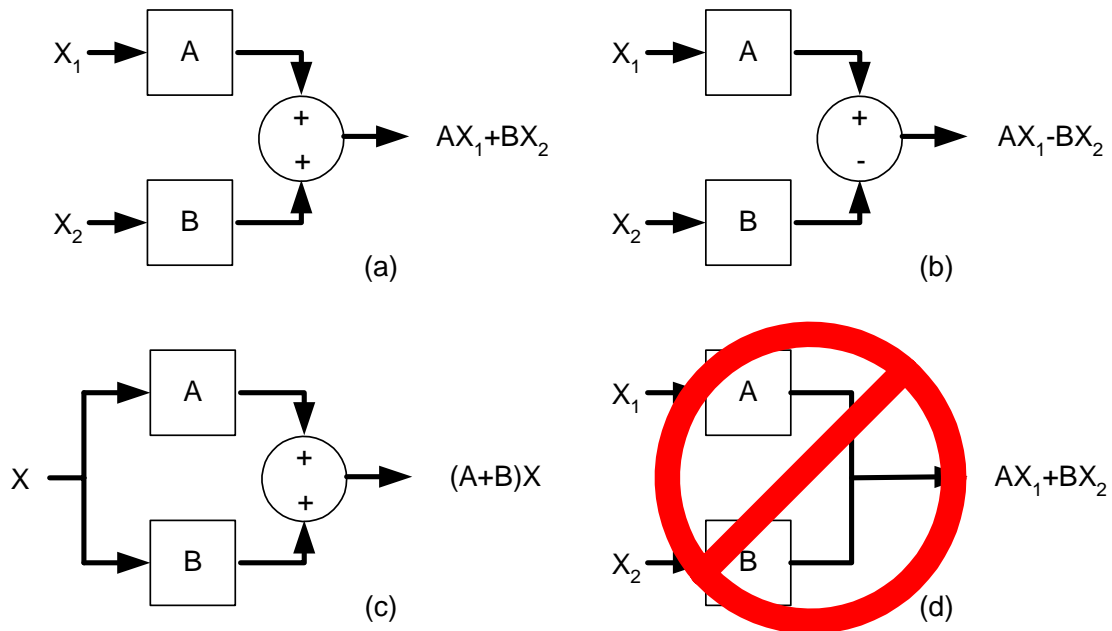


Figure 3: Three correct and one incorrect examples of block diagram addition. Figure (d) is incorrect because there is no summing junction connecting the outputs of blocks A and B so one does not know how to combine their outputs.

Figure 3b shows that subtraction is accomplished by changing one or more of the signs on the summing junction.

Figure 3d shows that block outputs **cannot** be combined without a summing junction.

Summing junctions are not limited to just two inputs.

Block Simplification

Block diagrams can be simplified. Multiple blocks and summing junctions can be reduced to a single block which is equivalent to the original diagram.

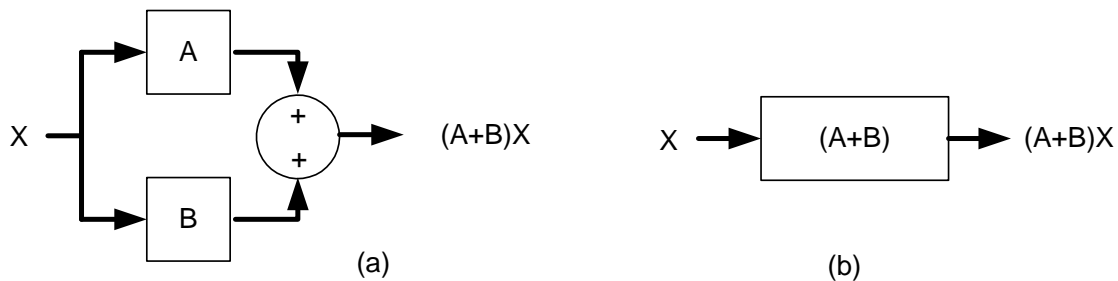


Figure 4: The diagram in (a) can be simplified to the diagram in (b) and they are equivalent.

Feedback Loops

The feedback loop is a very important block diagram structure. These will be studied extensively later. For now, it is enough to know how to simplify them to a single block.

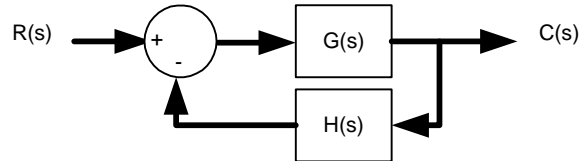


Figure 5: A negative feedback loop.

Figure 5 illustrates a negative feedback loop. The “feedback loop” refers to taking a portion of the output signal, $C(s)$, and combining it with the input signal $R(s)$, via the $H(s)$ block and the summing junction. “Negative” refers to the negative sign in the summing junction. A “positive feedback loop” is illustrated in Figure 6. Note the sign difference in the summing junction.

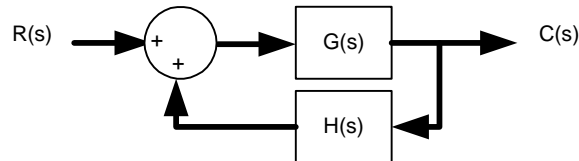
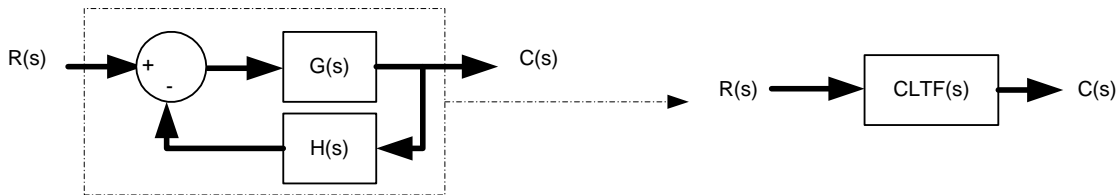


Figure 6: A positive feedback loop. Note the signs in the summing junction.

The block diagram in Figures 5 and 6 are fundamentally different from the erroneous block diagram depicted in Figure 3d because of the direction of the arrow on the output side.

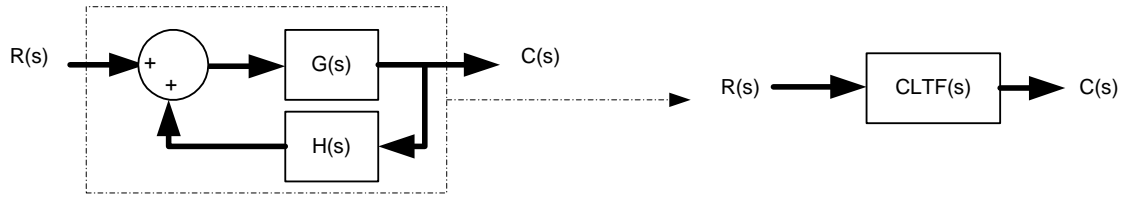
Positive and negative feedback loops can be reduced to a single block and function. This function is the “Closed Loop Transfer Function.”



$$CLTF_{negative}(s) = \frac{G(s)}{1 + G(s)H(s)}$$

Figure 7: A negative feedback loop reduced to a single block and function.

Lesson 1b: Block Diagrams



$$CLTF_{positive}(s) = \frac{G(s)}{1 - G(s)H(s)}$$

Figure 8: A positive feedback loop reduced to a single block and function.

Figures 7 and 8 give the CLTF for negative and positive feedback loops. Note that the sign in the denominator of the CLTF is the **opposite** of the sign in the summing junction.

Closed Loop Transfer Function Derivation

For the sake of completeness here is the derivation of the CLTF for a negative feedback loop. To help keep the derivation clear, all the (s) notations in the functions R, E, G, H, and C have been omitted. Remember these are all functions of the variable (s). The segment between the summing junction and G(s) has been designated E(s). E is for **Error** signal.

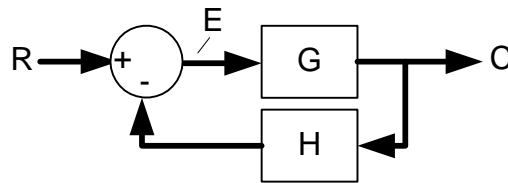


Figure 9: A negative feedback loop with the error signal, E, labeled.

The output C(s) is given by

$$C = G \cdot E .$$

The error signal E(s) is given by

$$E = R - H \cdot C .$$

Substituting the second equation into the firsts yields

$$C = G \cdot (R - H \cdot C) = G \cdot R - G \cdot H \cdot C .$$

Bringing C(s) to one side and separating out R(s)

$$C + G \cdot H \cdot C = (1 + G \cdot H) \cdot C = G \cdot R$$

$$C = \frac{G}{1 + G \cdot H} \cdot R .$$

Lesson 1b: Block Diagrams

Questions

Traditionally, the input function to a system is named $R(s)$ and the output is named $C(s)$. The functions $G(s)$ and $H(s)$ are arbitrary functions for particular blocks.

For each of the block diagrams calculate the output $C(s)$.

1)  $C(s)=$

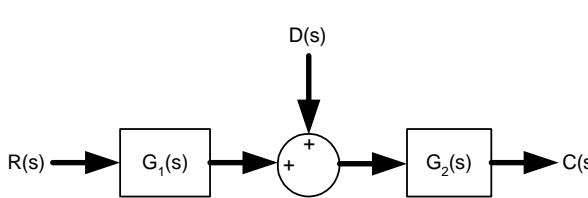
2)  $C(s)=$

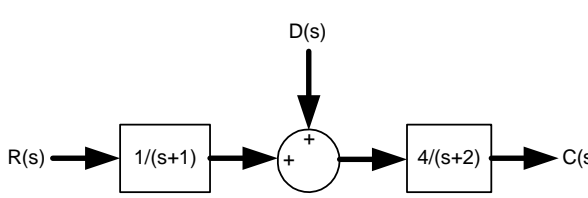
3)  $C(s)=$

4)  $C(s)=$

5)  $C(s)=$

6)  $C(s)=$

7)  $C(s)=$

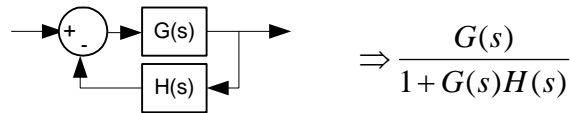
8)  $C(s)=$

Lesson 1b: Block Diagrams

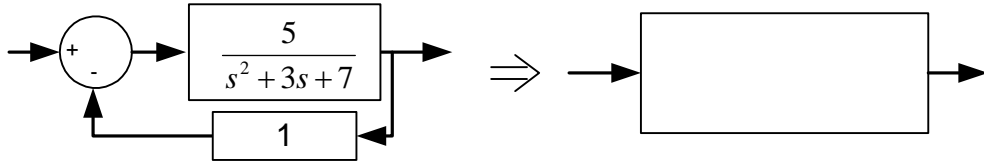
Find the closed loop transfer functions (CLTF) for the negative feedback loops below.

Your answer MUST be in the form $\frac{\text{polynomial}_1}{\text{polynomial}_2}$. For example: $\frac{5}{s^2 + 3s + 7}$.

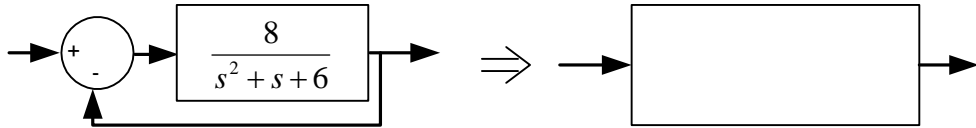
Do not use your calculator! Practice now, will save a lot of time later!



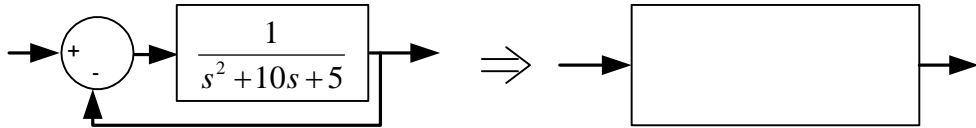
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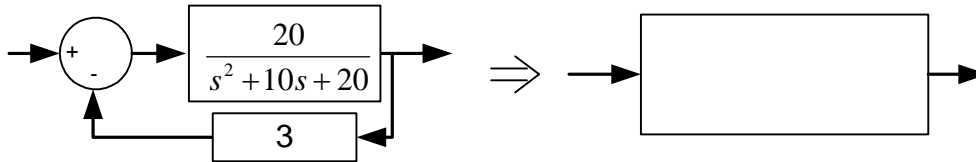
10)



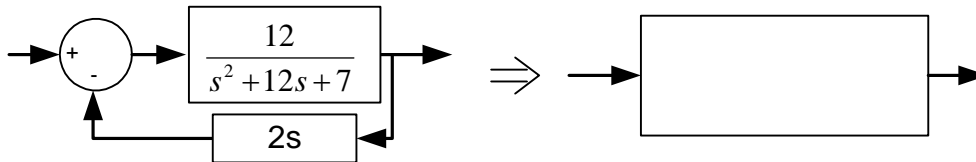
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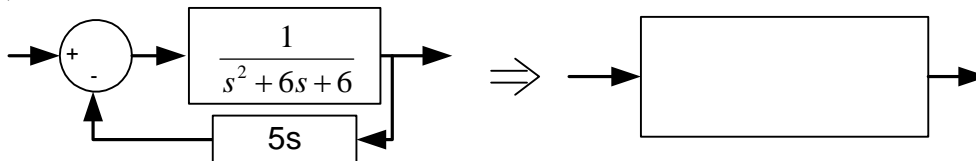
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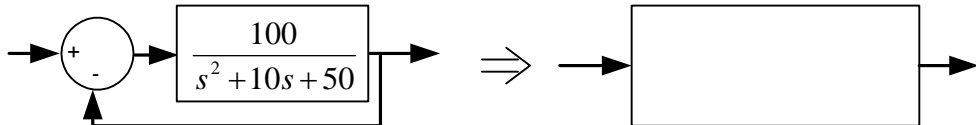
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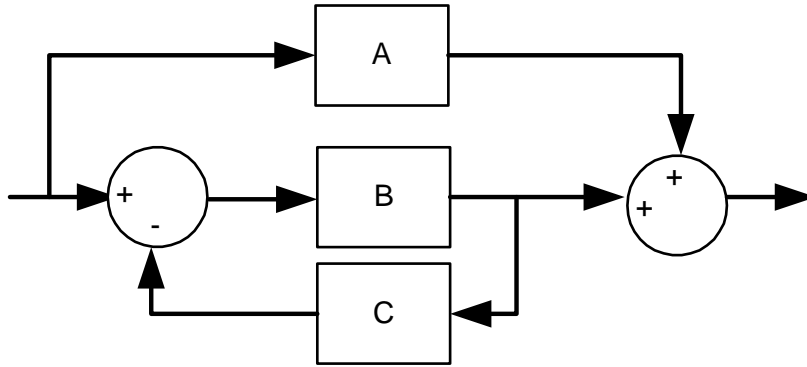


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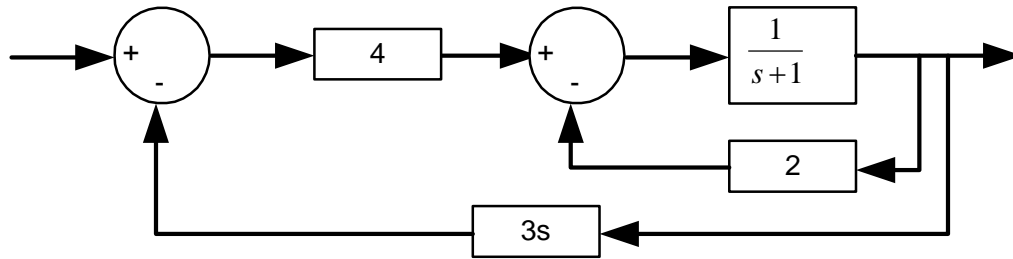
Lesson 1b: Block Diagrams

16) Reduce the following block diagram to a rational function. Sketching the intermediate block diagrams will be helpful. **Do not use your calculator.**



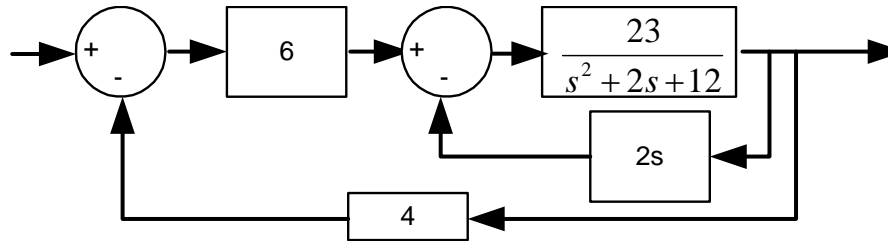
Lesson 1b: Block Diagrams

17) Reduce the following block diagram to a rational function. Sketching the intermediate block diagrams will be helpful. **Do not use your calculator.**



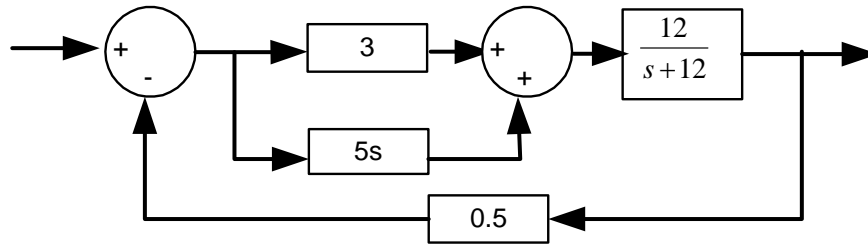
Lesson 1b: Block Diagrams

18) Reduce the following block diagram to a rational function. Sketching the intermediate block diagrams will be helpful. **Do not use your calculator.**



Lesson 1b: Block Diagrams

19) Reduce the following block diagram to a rational function. Sketching the intermediate block diagrams will be helpful. **Do not use your calculator.**



Frequency Domain & Building Complicated Wave Forms

Introduction and Overview

In this exercise you will compare the frequency domain to the time domain for several common wave forms. Figure 1 shows to the wave forms we will be examining.

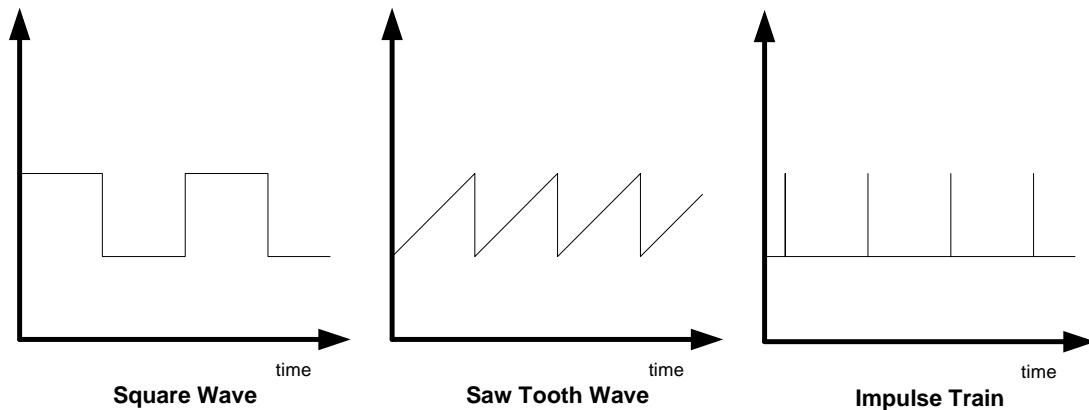


Figure 1: The ideal square wave, saw tooth wave, and an impulse function in the time domain.

These wave forms can be produced by summing a series of component sine or cosine waves. Table 1 lists the series which produces the different wave types.

Wave	Series
Square Wave	$\sin(t) + \frac{\sin(3 \cdot t)}{3} + \frac{\sin(5 \cdot t)}{5} + \frac{\sin(7 \cdot t)}{7} + \frac{\sin(9 \cdot t)}{9} + \dots$
Saw Tooth Wave	$\frac{\sin(2 \cdot t)}{2} + \frac{\sin(4 \cdot t)}{4} + \frac{\sin(6 \cdot t)}{6} + \frac{\sin(8 \cdot t)}{8} + \frac{\sin(10 \cdot t)}{10} \dots$
Impulse Train	$\cos(t) + \cos(2 \cdot t) + \cos(3 \cdot t) + \cos(4 \cdot t) + \cos(5 \cdot t) + \cos(6 \cdot t) + \dots$

Table 1: The series for a square wave, a saw tooth wave, and an impulse train.

Tool Setup and Use

The file **wavegen.m** is a MATLAB application which allows you to quickly plot a series of sine or cosine waves in both the time and frequency domains.

To run the application you must

1. Download the file **wavegen.m** to a directory for this week's work.
2. Open MATLAB
3. Set MATLAB's current directory to your directory for this week's work. Select the "Current Directory" option under the "Desktop" menu. When checked, the left hand pane of the MATLAB window will allow you to explore the file system. Go to your directory for this week.
4. Run the command *wavegen* at the MATLAB prompt. The command is case sensitive. **The command is wavegen, NOT wavegen.m.**

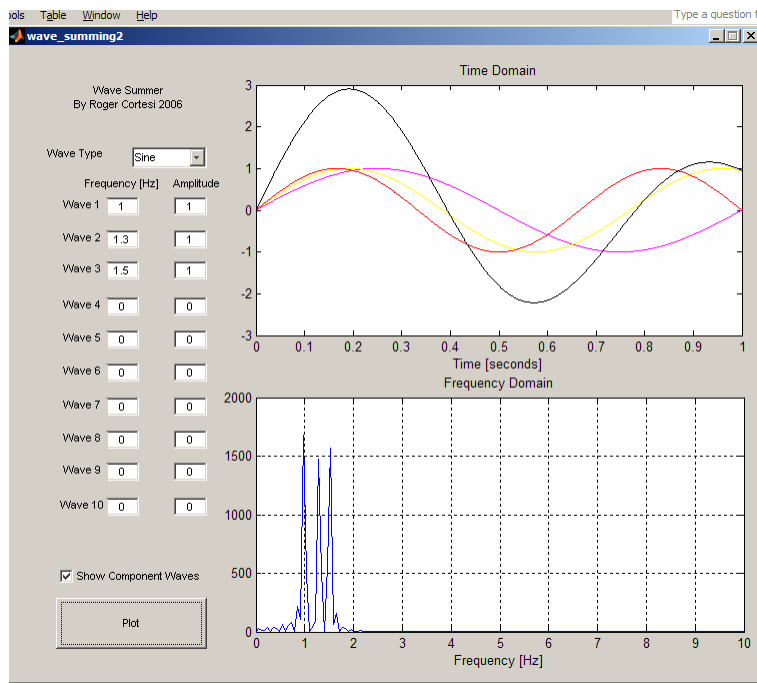


Figure 2: The ES360 wavegen application summing four sine waves.

The *wavegen* application will sum up to 10 sine or cosine waves. The sum of the component waves is the black curve in the time domain plot. This is the composite wave. The component waves may be displayed in the time domain plot (in color).

The frequency and amplitude must be entered as a decimal (not as a fraction).

The frequency domain plot displays the frequency spectrum of the composite wave.

The wave type menu sets all component waves are to either sine or cosine.

To sum less than 10 waves, set the amplitude to zero for the unused waves.

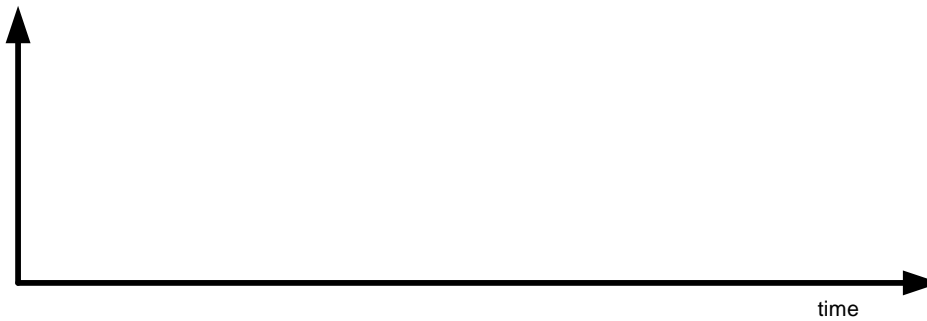
Questions

1) On the axis below sketch the **frequency** response for a single sine wave. Sketch what happens as you change the frequency and amplitude of the wave.

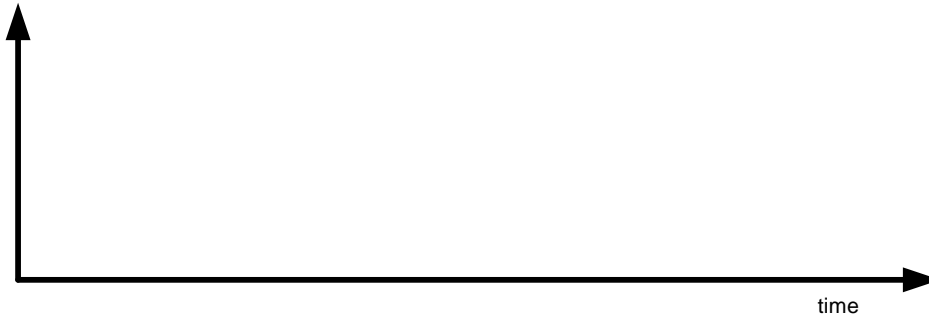


2) Using the *wavegen* command, plot each wave type in the time and frequency domains. Make a sketch of the plots on the axis below:

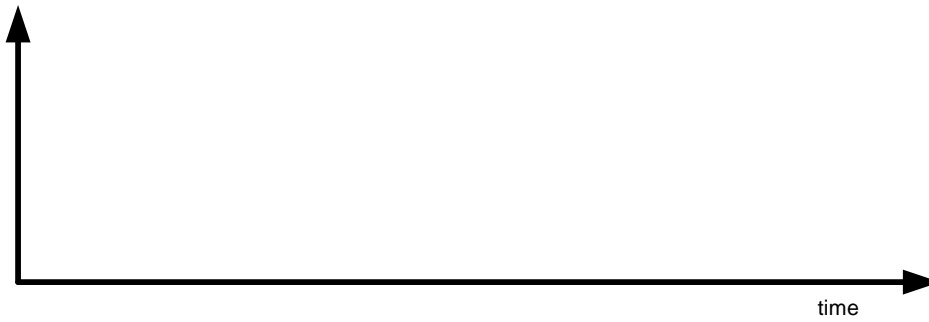
Square Wave



Saw Tooth Wave



Impulse



3) As you add more terms to the series (i.e. you sum more component waves) what happens to the composite wave in the time and frequency domains?

Basic Element Types

Introduction, the Mass Spring Damper

The three basic elements in system modeling are:

1. A kinetic energy storage element
2. A potential energy storage element
3. An energy dissipative element

Most real system can be modeled as combinations of these three elements.

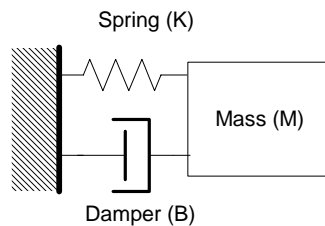


Figure 1: A simple mass spring damper system. This system has each of the three basic elements.

Figure 1 shows a simple mass spring damper system. The spring is the potential energy storage element, storing energy in its compression. The mass is the kinetic energy storage element, storing energy in its momentum. The damper (a source of friction) is the energy dissipative element.

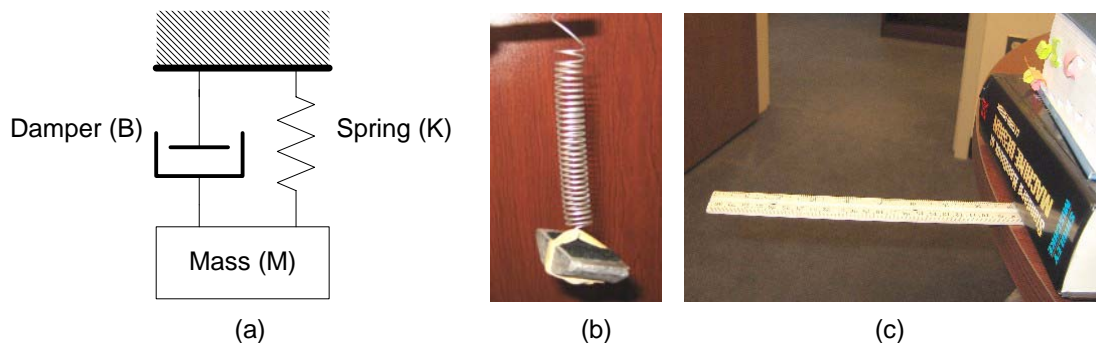


Figure 2: The mass spring damper model (a) can be applied to systems which clearly have separate mass spring damper components (b) and those that do not (c).

It is important to understand that these three elements are concepts, and not necessarily individual physical components of the system. For example, a mass hanging from a Slinky can be modeled using the above mass spring damper model. In this case the three elements are distinct physical parts of the system.

A cantilevered beam sticking out of a wall can also be modeled using the same three components. In this case there is only one physical part (the beam), but the beam exhibits characteristics of all three elements (mass, springiness, and friction).

In both cases applying a force to the system (hitting it with a hammer) will cause it to vibrate back and forth until the friction dampens out the oscillations.

Basic Elements in Other Energy Domains

The rotational, electrical, fluid and thermal¹ energy domains all have similar elements to the mechanical translational domain discussed above.

Domain	Kinetic Energy	Potential Energy	Dissipative
Mechanical (translational)	Mass	Spring	Damper
Mechanical (rotational)	Rotational Inertia	Spring	Damper
Electrical	Inductor	Capacitor	Resistor
Fluid	Fluid Mass	Fluid Capacitance (a reservoir)	Fluid Resistance
Thermal	Thermal Capacitance (Thermal Mass)		Thermal Resistance

Table 1: Basic element types for five different energy domains.

Energy Transfer Between Different Energy Domains

Real objects are normally composed of several different energy domains (mechanical, electrical, fluid, etc.). These different domains interact with each other by moving energy between them. In addition to the conceptual elements described in Table 1, there are many physical elements which transfer energy between the different energy domains.

Complete the table below.

Domain 1	Domain 2	Objects that Transfer Energy from Domain 1 to Domain 2
Rotational	Translational	
Rotational	Electrical	
Rotational	Fluid	
Fluid	Translational	
Electrical	Thermal	
Electrical	Rotational	
Rotational	Thermal	

¹ The behavior of thermal systems is slightly different from the other four energy domains.

The thermal capacitance of a system stores both kinetic and potential energy. The kinetic energy is stored in the translational motion of the atoms. The potential energy is associated with the intermolecular attractive and repulsive forces.

In the other systems the dissipative elements convert motion to heat via friction or electrical resistance. In the thermal system the dissipative element delays the propagation of heat in time and space.

Questions

In the images below label the energy storage elements (kinetic and potential) and the dissipative elements. Label any conversions between energy domains.

Hydro-electric Dam



Suspension Bridge



Mountain Bike



Squash Racquet and Ball



DC Electric Motor and Gear Box



Bungee Jumping



Solar Panels for Heating Water

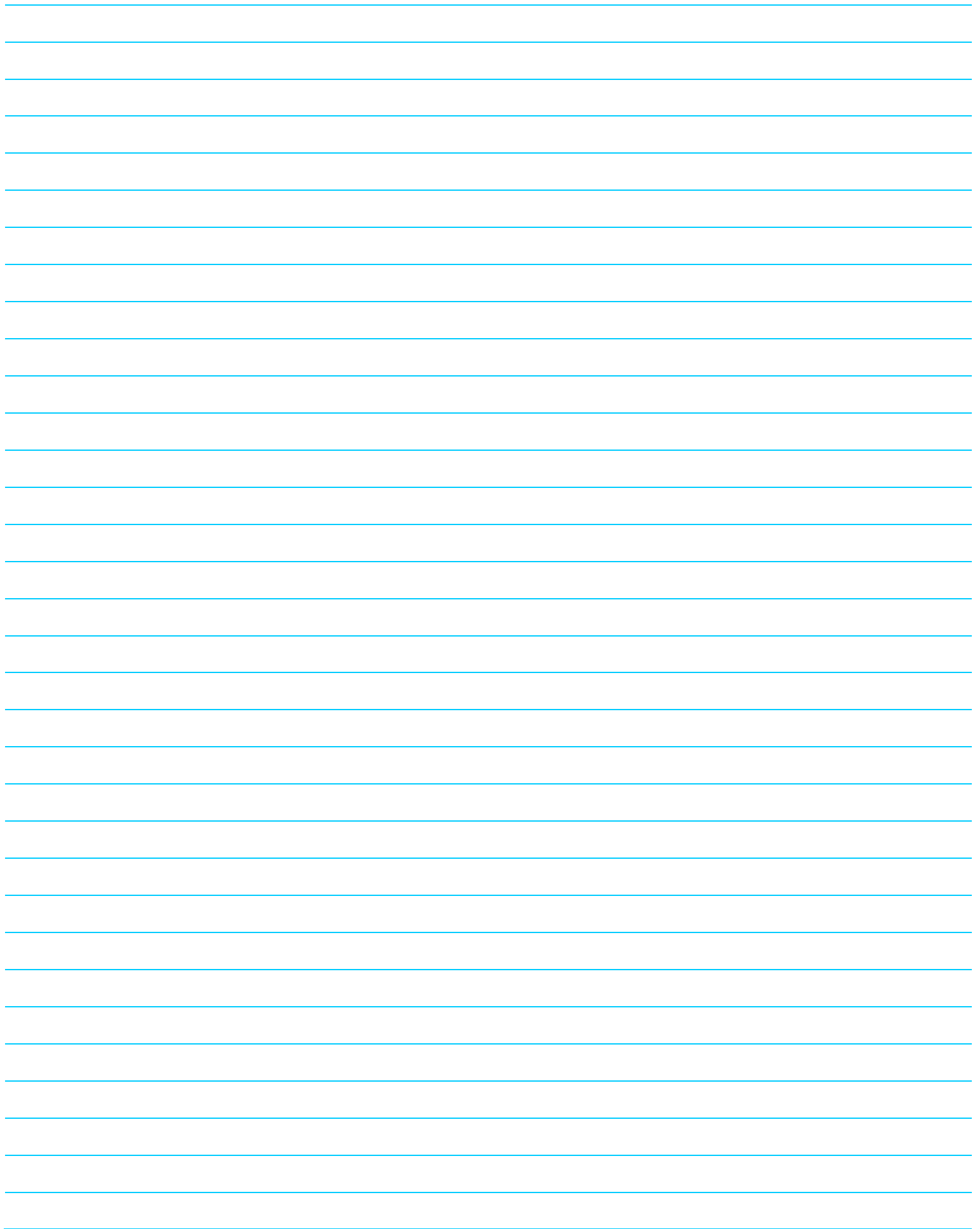


Tanqueray™



Steam Engine





Transfer Functions

Introduction and Overview

A transfer function is a model of a system or system component. It is the ratio of the output to the input **in the frequency domain**.

$$\text{TransferFunction} = \frac{\text{Output}}{\text{Input}} = G(s) = \frac{Y(s)}{X(s)}$$

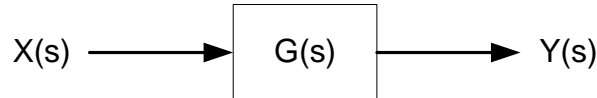


Figure 1: A transfer function $G(s)$ with an input $X(s)$ and an output $Y(s)$.

Transfer functions, their inputs, and their outputs are all functions of s , where s is frequency.¹

The Derivation of a Transfer Function

There are several methods for determining the transfer function for a system. In this course we will usually just state the transfer function. However, it is important to understand where these transfer functions come from. The following is a short derivation of the transfer function for a mass-spring-damper system.

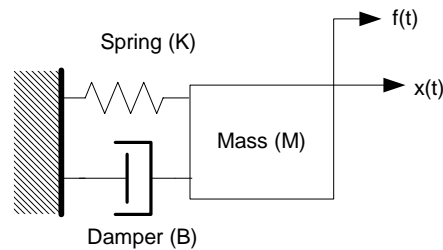


Figure 2: A simple mass-spring-damper system.

A transfer function describes the relationship between a particular input to a particular output. A single system can have multiple transfer functions, each relating a different input to a different output. In this mass-spring-damper example we will consider the possible inputs and outputs as²:

- Input: The force applied to the mass as a function of time $f(t)$.
- Output: The position of the mass as a function of time $x(t)$.

This system has three elements, the mass, the spring, and the damper. Each of these elements have a different relationship between force and position.

¹ Strictly speaking, s contains more information than frequency alone. This will be covered later in the course.

² Other possible inputs and outputs for this mass-spring-damper system could include its velocity or acceleration.

Lesson 2b: First Order Transfer Functions and the Step Response

Element	Force Displacement Relationship	Force is proportional to...
Spring	$f(t) = k \cdot x(t)$ (Hook's Spring Law)	Position
Damper (friction)	$f(t) = b \cdot v(t) = b \cdot \frac{d}{dt} x(t)$	1 st Derivative of Position (Velocity)
Mass	$f(t) = m \cdot a(t) = m \cdot \frac{d}{dt} v(t) = m \cdot \frac{d^2}{dt^2} x(t)$	2 nd Derivative of Position (Acceleration)

Table 1: The force displacement relationships for a mass, spring, and damper.

The sum of all the forces on the mass is

$$m \cdot \frac{d^2}{dt^2} x(t) + b \cdot \frac{d}{dt} x(t) + k \cdot x(t) = f(t).$$

This is a second order differential equation describing the relationship between force and position for the system *in the time domain*.

We can convert this equation to an algebraic equation in the frequency domain by taking the Laplace transform of it.

Time Domain Functions	Frequency Domain Functions
$x(t)$	$X(s)$
$\frac{d}{dt} x(t)$	$s \cdot X(s)$
$\frac{d^2}{dt^2} x(t)$	$s^2 \cdot X(s)$

Table 2: Some functions in the time domain and their Laplace transforms in the frequency domain.³

Taking the Laplace transform of each term in the differential equation yields

$$m \cdot s^2 \cdot X(s) + b \cdot s \cdot X(s) + k \cdot X(s) = F(s).$$

Simplifying the equation by factoring the X(s) terms yields

$$(m \cdot s^2 + b \cdot s + k) \cdot X(s) = F(s).$$

Recall that a transfer function is the ratio of the output over the input. In this case, the output parameter is X(s) and the input parameter is F(s). The transfer function for the mass-spring-damper system is

$$G(s) = \frac{X(s)}{F(s)} = \frac{1}{m \cdot s^2 + b \cdot s + k}.$$

³ The Laplace transform of $\frac{d}{dt} x(t)$ is actually $s \cdot X(s) - x(0)$, where x(0) is the initial condition of the function in the time domain. Transfer functions assume that all initial conditions are zero. So the initial conditions have been omitted from Table 2.

A System's Order

A system's order is physically determined by the number of independent energy storage elements within the system. This corresponds to the highest power of s in the transfer function's denominator. For example:

$$\frac{1}{s+1}$$

is a transfer function for a 1st order system with only a single energy storage element.

$$\frac{1}{s^2 + 2s + 3}$$

is a transfer function for a 2nd order system with two energy storage elements. And

$$\frac{1}{s^3 + 7s^2 + 4s + 2}$$

is a transfer functions for a 3rd order system with three energy storage elements.

A Transfer Function's Response

A transfer function is used to determine how a system will respond when a specific input is applied to it. There are a wide variety of possible inputs to a system. Several are shown in Figure 3.

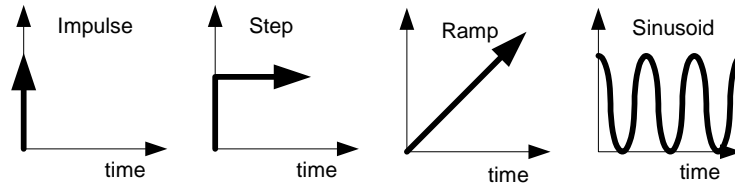


Figure 3: Four examples of input types in the time domains.

We can examine the system's response in either the *frequency domain* or the *time domain*. In this course, we will only be interested in the time response in the time domain.

Remember, the transfer function for a system is a model in the frequency domain. Therefore, any input in the time domain must be converted to the frequency domain before it can be applied to the transfer function. The calculated response is in the frequency domain, so it must be converted to the time domain to get the time response.

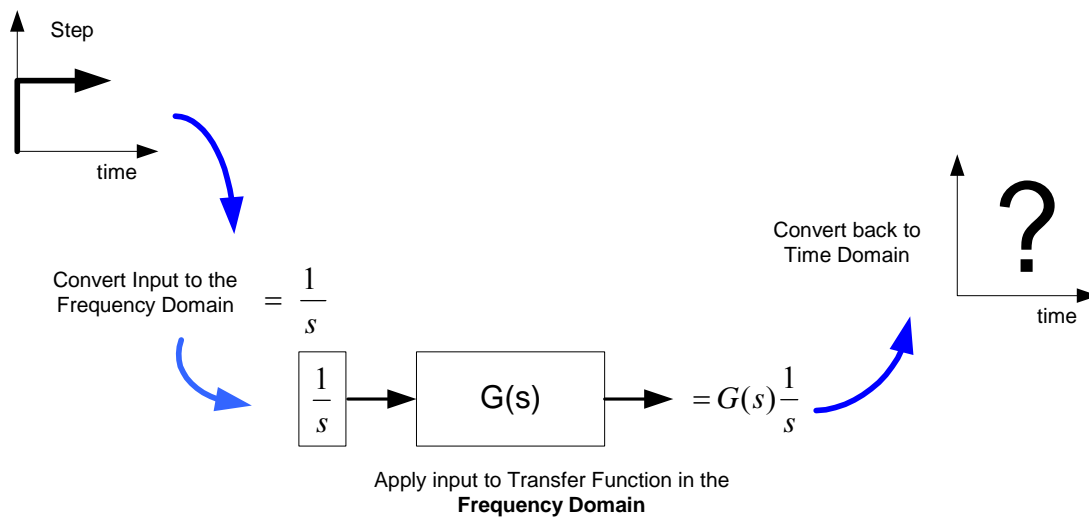


Figure 4: The basic process for finding the time response of a system using transfer functions and the frequency domain.

Functions are converted from the time domain to the frequency domain using the Laplace transform. Taking the Laplace transform of a system changes the problem from calculus to algebra, making it much easier to solve. For example, solving a 5th order polynomial is trivial compared to solving a 5th order differential equation.

First Order Transfer Functions and the Step Response

One of the simplest transfer functions is the first order transfer function. It has the general form

$$G(s) = \frac{a}{s + \sigma}.$$

A very common input type is the unit step. The unit step has a value of zero at times less than zero. At time equal to zero the unit step function changes to a value of 1.

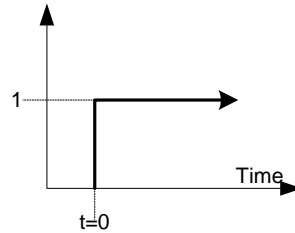


Figure 5: The unit step function.

Figure 5 depicts the unit step function in the *time domain*. To apply the unit step function as an input to a transfer function it needs to be converted to the *frequency domain*. The Laplace transform makes this conversion. The Laplace transform of a unit step is

$$\frac{1}{s}.$$

Recall that the transfer function is the ratio of the system's output over its input. Multiplying the transfer function by the input gives the output. This is very similar to performing unit analysis.

$$\text{input} \cdot \frac{\text{output}}{\text{input}} = \text{output}.$$

Applying this idea for a unit step input and the first order transfer function yields an output of

$$\frac{a}{s + \sigma} \cdot \frac{1}{s} = \frac{a}{s \cdot (s + \sigma)}.$$

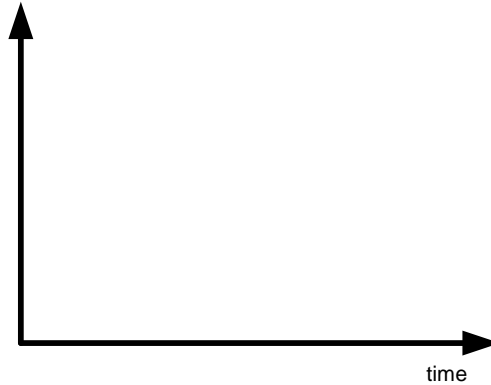
This output response is in the frequency domain (because it is a function of s). Take the inverse Laplace transform of the output to convert it to the time domain. The inverse Laplace transform for this example is

$$g(t) = \frac{a}{\sigma} \left(1 - e^{-\sigma t} \right).$$

We have deliberately skipped the details of taking the Laplace transform and the inverse Laplace transform. With a little practice, is it possible to quickly determine the important parameters of the *time response* directly from the system's transfer function *in the frequency domain*.

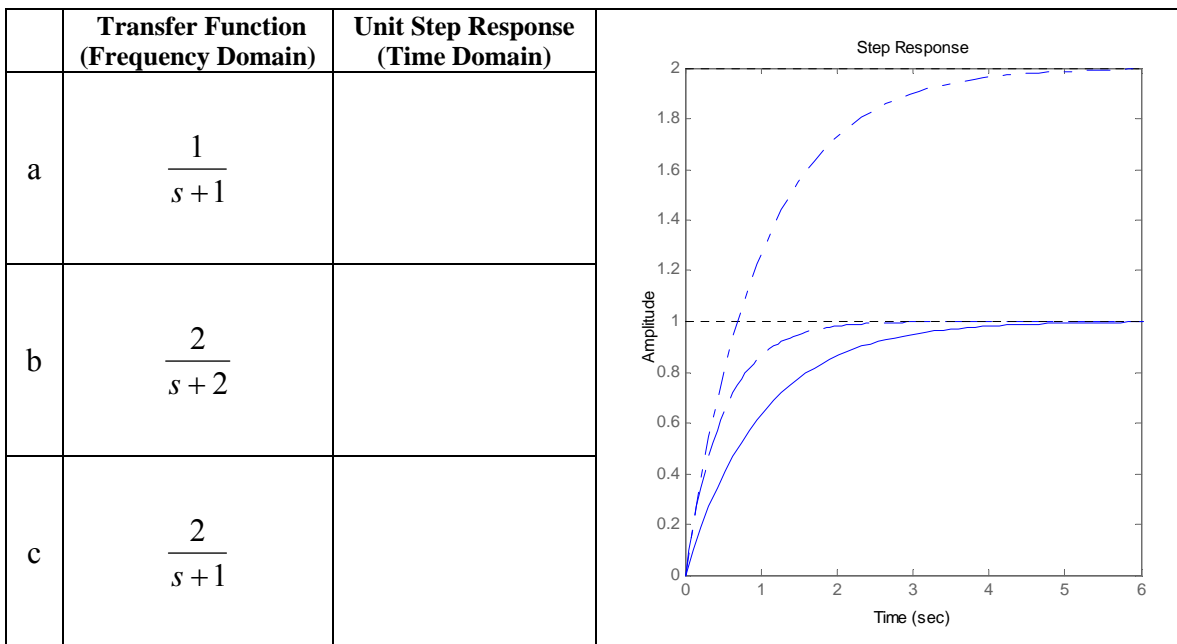
Lesson 2b: First Order Transfer Functions and the Step Response

1) Sketch a plot of $1 - e^{-t}$ on the axes below. Your calculator may be helpful.



All first order transfer functions have this shape as their step response!

2) Write the equation for the step response (in the time domain) for the following first order transfer functions. Label each curve on the plot with its corresponding transfer function. Your calculator may be helpful in correlating each curve with its transfer function.



Lesson 2b: First Order Transfer Functions and the Step Response

The step response of a first order system can be measured by two parameters.

- **Settling Time:** This is the length of time it takes the system to reach its final value.
- **DC Gain:** This is the value of the transfer function as frequency (s) approaches zero. Frequency approaching zero is equivalent to time approaching infinity. It is the ratio of the steady state output over the steady state input (the step magnitude).

Settling Time

Mathematically the system approaches its final value asymptotically. Theoretically it never actually reaches the final value. For practical purposes, the system is considered at its final value when it is within 2% of the asymptotic value⁴.

Recall, the general form of the step response in the time domain is given by

$$g(t) = \frac{a}{\sigma} (1 - e^{-\sigma t}).$$

3) How are the values for DC gain and the settling time specified in this equation?

DC Gain:	Settling Time:
-----------------	-----------------------

In a first order transfer function $\frac{1}{\sigma}$ is the *time constant* of the system. The value of σ determines how quickly the system will approach its final value.

4a) Find the settling time (T_s) in terms of σ by solving the following equation for t . Show each step!

$$2\% = e^{-\sigma t}$$

$T_s =$

Since these calculations are approximations of the system's response, the answer above is rounded to

$$T_s = \frac{4}{\sigma}$$

4b) After one time constant has elapsed, $t = \frac{1}{\sigma}$, how close is a first order system to its final value? Hint: Solve $1 - e^{-\sigma t}$.

Percent of System's Final Value After One Time Constant =
--

⁴ The 2% tolerance is common for measuring the performance of mechanical systems. In electrical engineering the performance tolerance on the final value is often 0.7% percent.

DC Gain

A DC signal has a frequency of zero. Gain means multiplication. A system's DC gain is the value of the transfer function at a frequency (s) of zero. This is the value the transfer function multiplies the input by at a frequency of zero. Recall

$$\text{Frequency} = \frac{1}{\text{Time}}.$$

Frequency approaching zero is equivalent to time approaching infinity. This means the DC gain of a transfer function is also the amount the input will be multiplied by as time approaches infinity.

Figure 6 shows the unit step input and the response for the transfer function

$$G(s) = \frac{2}{s+1}.$$

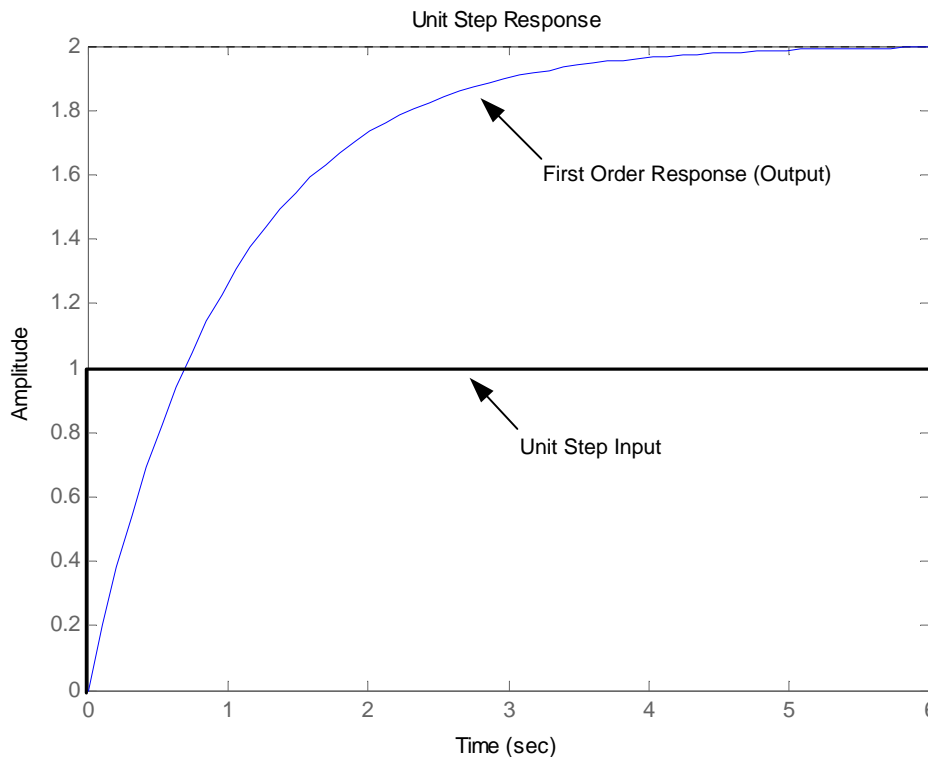


Figure 6: The unit step input and first order response for a transfer function with a DC gain of 2.

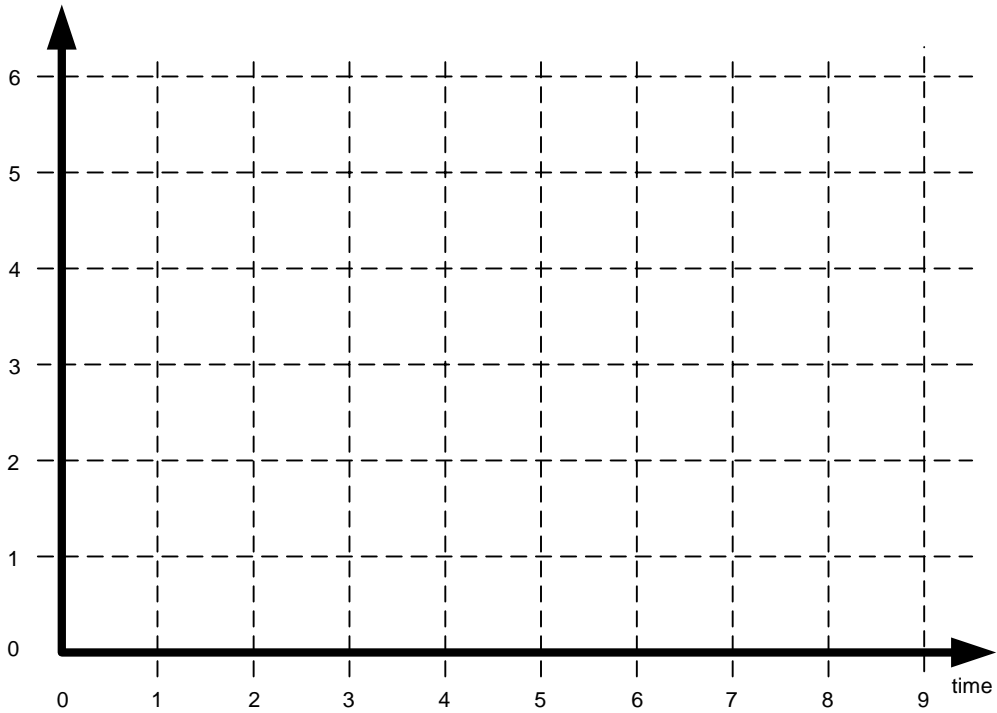
Figure 6 illustrates the transfer function is multiplying the input by two at steady state. Therefore, the transfer function has a DC gain of two.

Lesson 2b: First Order Transfer Functions and the Step Response

5) Calculate the DC gain and settling time for the following first order transfer functions.

	Transfer Function	DC gain (value at s=0)	Settling Time (Ts)
a	$\frac{1}{s+1}$		
b	$\frac{2}{s+1}$		
c	$\frac{2}{s+2}$		
d	$\frac{3}{s+0.5}$		
e	$\frac{32}{s+16}$		
f	$\frac{8}{s+8}$		
g	$\frac{s}{s+8}$		

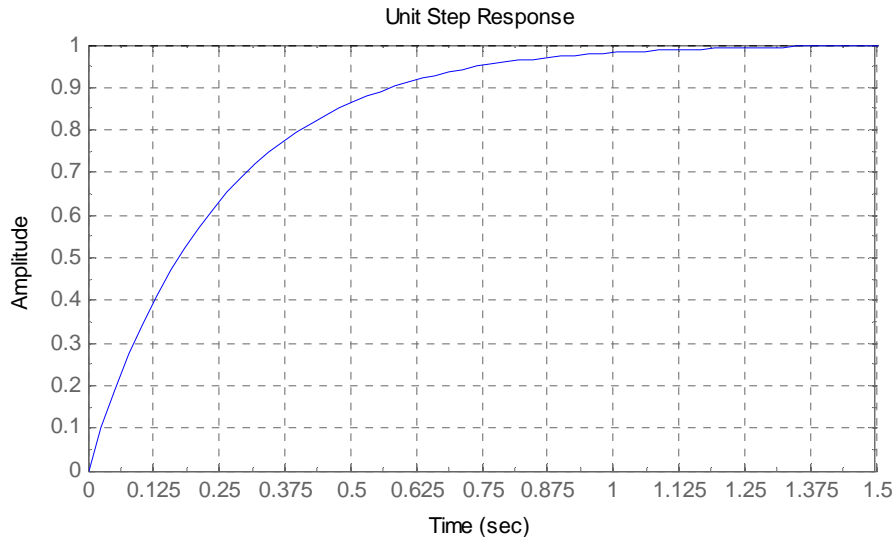
6) Plot the unit step response for transfer functions 5d, 5e, and 5f on the axes below.



Lesson 2b: First Order Transfer Functions and the Step Response

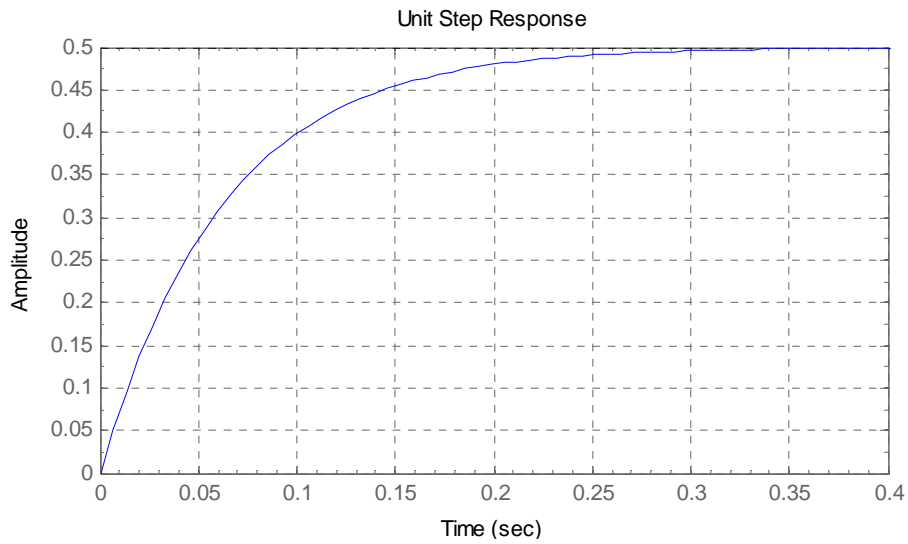
7) Write the first order transfer function for each plot of the **unit** step response.

a)



G(s) =

b)



G(s) =

Systems with First Order Transfer Functions in 5 Energy Domains

8) Specify the input and output parameters for each transfer function. Solve for their settling time and DC gain.

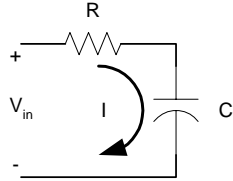
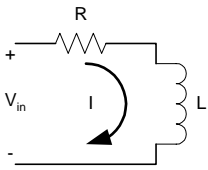
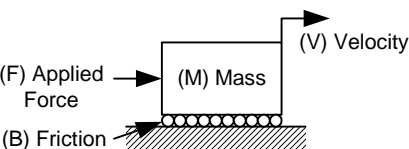
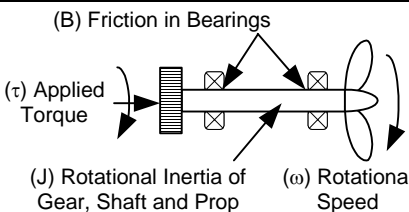
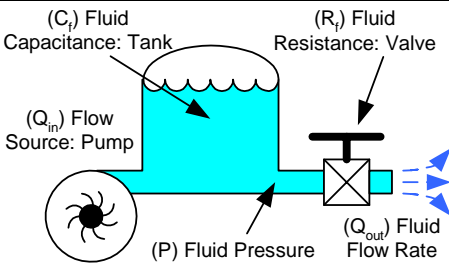
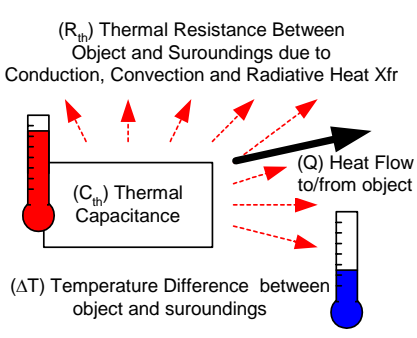
System	Transfer Function	Ts	DC Gain
	$G(s) = \frac{I(s)}{V(s)} = \frac{\frac{1}{R} s}{s + \frac{1}{RC}}$		
	Input: Output:		
	$G(s) = \frac{I(s)}{V(s)} = \frac{\frac{1}{L}}{s + \frac{R}{L}}$		
	Input: Output:		
	$G(s) = \frac{Vel(s)}{F(s)} = \frac{\frac{1}{M}}{s + \frac{B}{M}}$		
	Input: Output:		
	$G(s) = \frac{\omega(s)}{\tau(s)} = \frac{\frac{1}{J}}{s + \frac{B}{J}}$		
	Input: Output:		
	$G(s) = \frac{P(s)}{Q_{in}(s)} = \frac{\frac{1}{C_f}}{s + \frac{1}{R_f C_f}}$		
	Input: Output:		
	$G(s) = \frac{\Delta T(s)}{Q(s)} = \frac{\frac{1}{C_t}}{s + \frac{1}{R_t C_t}}$		
	Input: Output:		
	$G(s) = \frac{T_{obj}(s)}{T_{amb}(s)} = \frac{\frac{1}{R_t C_t}}{s + \frac{1}{R_t C_t}}$		
	Input: Output:		

Table 3: First order systems and their transfer functions in five energy domains.

Lesson 2b: First Order Transfer Functions and the Step Response

For the systems pictured in questions 9 through 13, write a first order transfer function describing them and answer the associated questions.

9)



Figure 7: A skydiver has mass and friction.

a)

G(s)=

b) Using the transfer function, show what happens to the skydiver's velocity as friction increases.

c) Does mass of the skydiver affect the DC gain of the transfer function?

d) How does increasing the mass of the skydiver affect his final speed? Why?

e) Is the magnitude of "terminal velocity" affected by the transfer function's DC gain or settling time?

Lesson 2b: First Order Transfer Functions and the Step Response

10) Reciprocating engines (like the one in a car or steam engine) produce varying amounts of torque. They produce the most torque when a cylinder is firing. When no cylinders are firing, no torque is generated. A flywheel is used to smooth out the rotary motion produced by the engine. The momentum of the flywheel keeps the engine rotating when a cylinder is not firing.

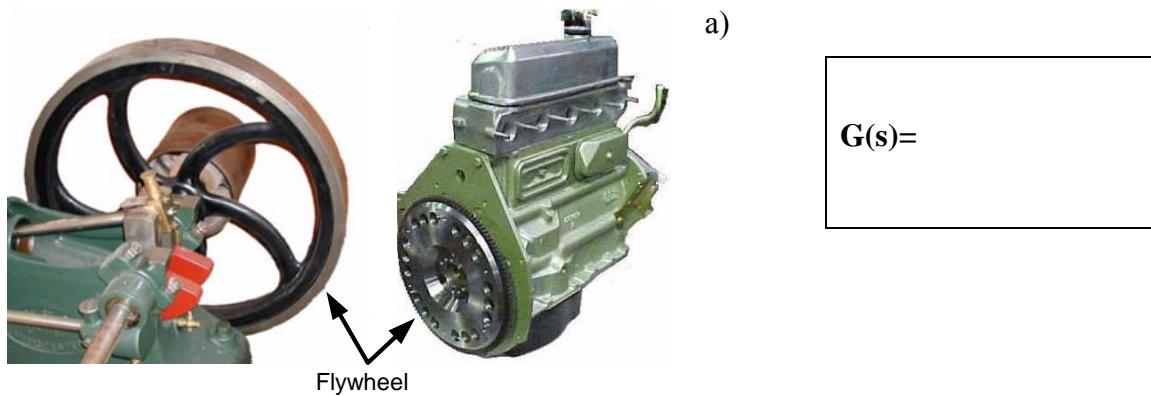


Figure 8: Flywheels are used to smooth the output of external (left) and internal (right) combustion engines.

b) Which parameter in the transfer function determines how well the flywheel smooths out the rotary motion of the engine? Does this parameter change the steady state speed of the engine (i.e. the DC gain of the transfer function)?

c) Is it desirable to have a long or short settling time for a flywheel? Why?

d) What are two methods of adjusting the settling time of the flywheel? Why is one of these methods undesirable in an engine?

Lesson 2b: First Order Transfer Functions and the Step Response

11) The nuclear fuel in a reactor generates heat. This heat flows from the fuel through the cladding and to coolant. The coolant transports this heat to the steam generators where it is used to make steam and do work.

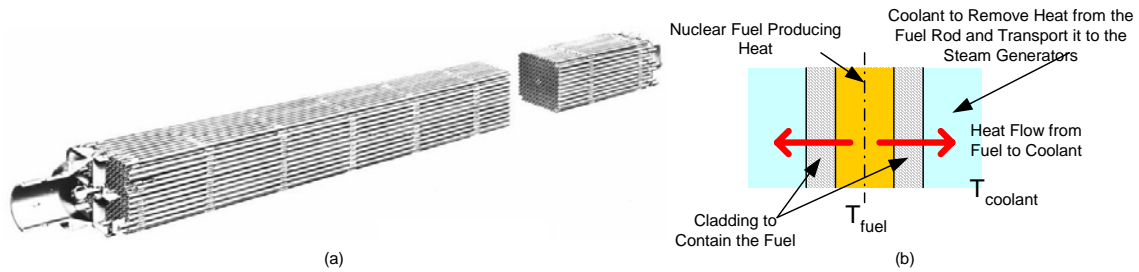


Figure 9: Nuclear fuel elements for the cargo ship *NS Savannah* (a), the only civilian nuclear vessel ever constructed. It was not commercially viable. A simplified diagram of the heat flow from the fuel to the coolant (b).

a) Hint: This question is about heat flow.

$G(s)=$

b) Which parameter in the transfer function will determine the steady state temperature difference between the fuel and the coolant? How can this temperature difference be minimized?

c) Which parameter(s) in the transfer function determine how quickly the temperature difference will occur when the reactor starts generating heat? What physically determines the value of the parameter(s).

Lesson 2b: First Order Transfer Functions and the Step Response

12) Large metal objects, like ships, take a significant amount of time to heat up and cool down. The environment around the ship will heat up and cool down much more quickly.

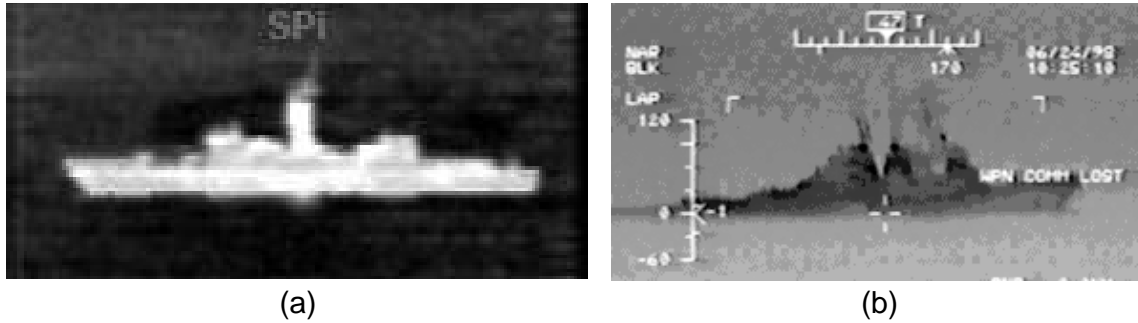


Figure 10: Thermal images of a ship several hours after sunset (a) and of another ship several hours after sunrise (b).

Figure 10a shows a thermal image of a ship several hours after sunset. The ocean and atmosphere have cooled down faster than the ship. The ship will remain warmer than the environment for many hours after sunset. The thermal imaging system indicates this by displaying the ship as lighter than its surroundings.

Throughout the night, the ship’s temperature cools down to match the ambient temperature. Figure 10b shows a thermal image of a ship a few hours after sunrise. The environment has warmed up much faster than the ship. The thermal imaging system indicates this by displaying the ship as darker than its surroundings.

a) What type of ships are shown in Figures 10a and 10b?

Ship Type 10a:	Ship Type 10b:
-----------------------	-----------------------

b) Write a first order transfer function which describes how the temperature of the ship responds to the changes in ambient temperature.

G(s) =

c) What is the ship’s temperature in steady state? How is this indicated by the transfer function?

d) How much heat is flowing to/from the ship if it is at ambient temperature?

e) Does increasing the thermal mass (capacity) of the ship change the settling time for the temperature of the ship? Why?

f) Does increasing the thermal resistance of the ship change the settling time for the temperature of the ship? Why?

f) Does the thermal resistance change the final temperature of the ship? Why?

Lesson 2b: First Order Transfer Functions and the Step Response

13) Water towers are often used as a pressure source for municipal water systems. A pump supplies the water to keep them full.



Figure 11: A water tower provides a fluid reservoir at a pressure for the town's water system.

a)

G(s) =

b) In the municipal water system, what physically determines the two parameters of the system's transfer function?

c) If the flow to the water tower is stopped, what happens to the pressure of the municipal water supply? Explain using the transfer function.

d) During commercial breaks in the Super Bowl the system pressure of the municipal water system will drop dramatically. Explain why in terms of the transfer function.

e) If water departments want to minimize the pressure drop during commercial breaks, which parameter of the system should they improve? Why can they not "improve" the other parameter?

Input Types

We will examine 4 common signals and apply them to a range of transfer functions.

These signals are similar to the three wave types introduced in Lesson 1c.

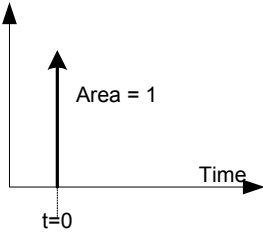
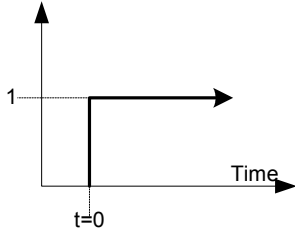
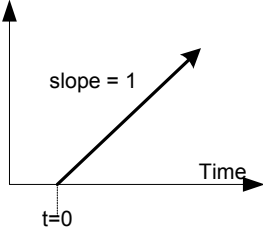
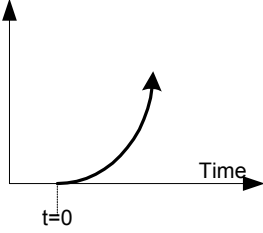
		Time Domain	Freq Domain	Notes and Example
Unit Impulse		$\delta(t)$	1	The unit impulse function is infinitely high and narrow such that the area under the curve is 1. Example: A short duration transient, from striking an object with a hammer.
Unit Step		$u(t)$	$\frac{1}{s}$	At time 0 the function $u(t)$ changes value from 0 to 1. Example: Applying a constant voltage starting at time 0, by turning a switch on.
Unit Ramp		t	$\frac{1}{s^2}$	At time 0 the unit ramp function starts increasing with a slope of 1. Example: The position of an object vs. time for an object moving at a constant velocity.
Unit Parabola		t^2	$\frac{1}{s^3}$	At time 0 the unit parabolic function starts increasing as a function of time squared. Example: The position of an object vs. time for an object moving with constant acceleration (falling).

Table 1: Four basic input types and their representations in the time and frequency domains.

Table 1 shows the **unit** form of all these signals. In the unit form their key parameter has a value of 1. Real signals usually have values other than 1. In this case the unit function is multiplied by the magnitude of the signal. For example:

- a) 5 volts is applied to a circuit, by closing a switch at time 0. This input signal would be described as 5 times the unit step. $5u(t)$ in the time domain or $\frac{5}{s}$ in the frequency domain.
- b) A hammer strikes an object with force of 10N. This input signal would be described as 10 times the unit impulse function. $10\delta(t)$ in the time domain or 10 in the frequency domain.

The Final Value Theorem

The final value theorem is a fast way of calculating the steady state value of a transfer function for a specific input. The final value theorem (FVT) states

$$Value_{SteadyState} = \lim_{s \rightarrow 0} s \cdot input(s) \cdot G(s).$$

Where $G(s)$ is the transfer function and the input are specified **in the frequency domain**. Don't forget the additional s term!

Figure 1 illustrates the output for a unit step applied to the transfer function

$$G(s) = \frac{5}{s^2 + 3s + 1}.$$

Initially the output is **changing**, this is called the **system's transient response**. Once the output has stopped changing, at about 7 seconds in this example, the system has reached its steady state. This is called the **steady state response** or the system's final value.

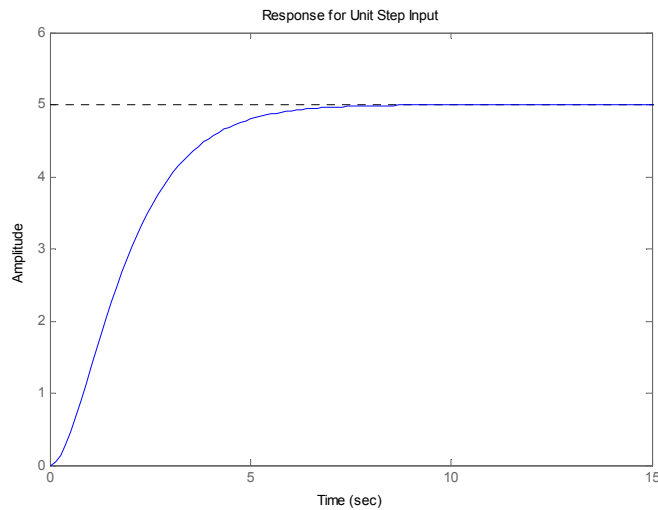


Figure 1: The output of a unit step applied to the example $G(s)$ vs. time. After about 7 seconds the output is no longer changing and the system has reached steady state (or its final value).

Using the final value theorem to calculate the system's steady state response yields.

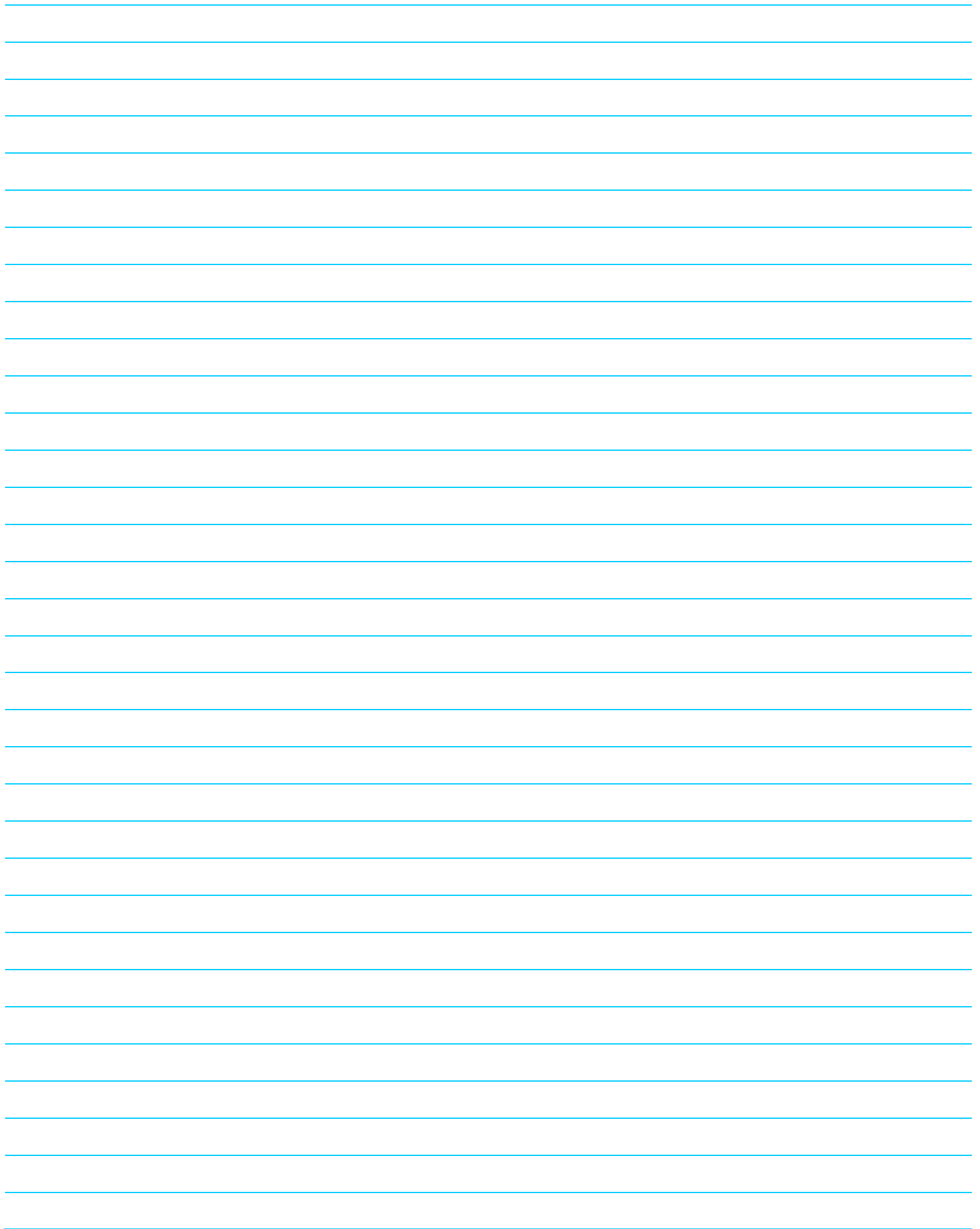
$$\lim_{s \rightarrow 0} s \cdot \frac{1}{s} \cdot \frac{5}{s^2 + 3s + 1} = 5.$$

The first s term and the $\frac{1}{s}$ cancel each other out. The s terms in the denominator of the transfer function are set to zero, leaving $1 \cdot \frac{5}{1} = 5$.

Final Value Theorem Exercise

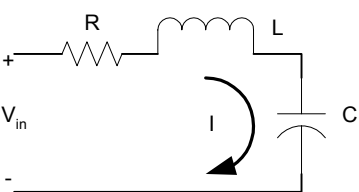
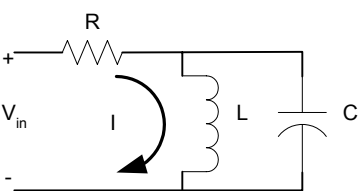
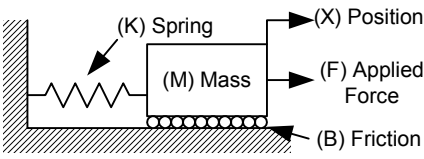
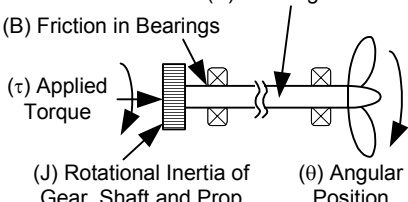
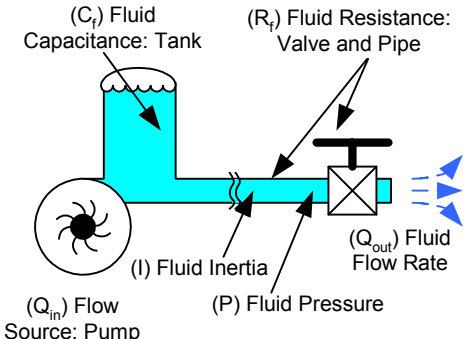
For each of the inputs and transfer functions below find the system's steady state value using the final value theorem. Additionally, state the type of input being applied to the system or give its function in the frequency domain.

	Input	Transfer Function	TF's DC Gain	Final Value	Input Type or Freq Domain
	$\frac{1}{s}$	$\frac{4}{s^2 + 2s + 2}$	2	2	unit step
	unit ramp	$\frac{3}{s^2 + 2s + 3}$	1	∞	$\frac{1}{s^2}$
a	$\frac{5}{s}$	$\frac{15}{s^2 + s + 3}$			
b	$\frac{4}{s^2}$	$\frac{12}{s^2 + 3s + 4}$			
c	unit step	$\frac{6}{s^2 + 2s + 3}$			
d	5	$\frac{6}{s^2 + 2s + 3}$			
e	$\frac{1}{s^2}$	$\frac{6s}{s^2 + 2s + 3}$			
f	$\frac{1}{s^3}$	$\frac{21}{s^2 + 2s + 3}$			
g	unit impulse	$\frac{25}{s^2 + 2s + 5}$			
h	4	$\frac{35}{s^2 + 5s}$			
i	step of magnitude 5	$\frac{12}{s^3 + 2s^2 + 3s + 4}$			
j	impulse of magnitude 6	$\frac{5}{s^2 + 5s}$			
k	$\frac{0.25}{s^3}$	$\frac{12s^2}{s^2 + s + 1}$			



Second Order Systems

Second order systems are systems with 2 energy storage elements. The table below illustrates second order systems in four different energy domains. For each system specify the energy storage elements (kinetic and potential energy) and the dissipative elements (DE). For each system's transfer function specify the input and the output.

System	Transfer Function	Elements
	$G(s) = \frac{I(s)}{V(s)} = \frac{\frac{1}{L} s}{s^2 + \frac{R}{L} s + \frac{1}{LC}}$	KE: PE: DE:
	$G(s) = \frac{I(s)}{V(s)} = \frac{\frac{1}{R} s^2 + \frac{1}{RLC}}{s^2 + \frac{1}{RC} s + \frac{1}{LC}}$	KE: PE: DE:
	$G(s) = \frac{X(s)}{F(s)} = \frac{\frac{1}{M}}{s^2 + \frac{B}{M} s + \frac{K}{M}}$	KE: PE: DE:
<p>Shaft is long enough to allow twisting.</p> 	$G(s) = \frac{\theta(s)}{\tau(s)} = \frac{\frac{1}{J}}{s^2 + \frac{B}{J} s + \frac{K}{J}}$	KE: PE: DE:
 <p>A long length of pipe allows fluid to build up inertia.</p>	$G(s) = \frac{P(s)}{Q_{in}(s)} = \frac{\frac{1}{C_f} s}{s^2 + \frac{1}{R_f C_f} s + \frac{1}{I_f C_f}}$ $G(s) = \frac{Q(s)}{P(s)} = \frac{\frac{1}{I_f} s}{s^2 + \frac{R_f}{I_f} s + \frac{1}{I_f C_f}}$	KE: PE: DE:

The Second Order Responses

A first order system only has one response to a step input (Lesson 2b). A second order system has two energy storage elements. It is possible for energy to pass back and forth between them, in addition to being dissipated out of the system. This complexity allows for many different responses.

A second order system can exhibit 6 possible responses to a step input. These are illustrated in Figure 1. The type of response is determined by the relative size of the energy storage elements and the amount of energy dissipation (friction) in the system.

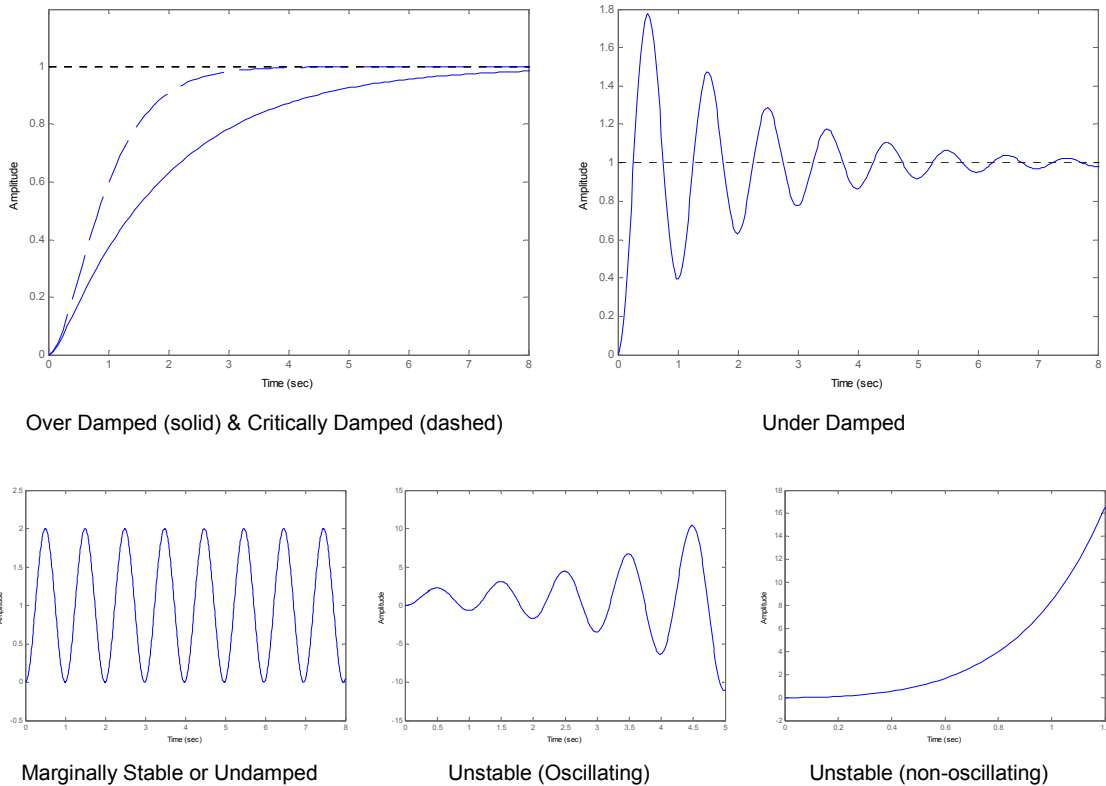


Figure 1: The six possible time responses for a 2nd order system.

1a) Which of these 6 cases has the most friction, zero friction, and “negative friction”?

1b) Do any of the cases ever reach an exact final value?

The Characteristic Equation

A second order system's transfer function specifies which of the 6 responses the system will exhibit. The response is determined by the transfer function's **characteristic equation**.

The characteristic equation is the denominator of the transfer function set equal to zero.

For example, the transfer function for a mass-spring-damper is

$$G(s) = \frac{\frac{1}{M}}{s^2 + \frac{B}{M}s + \frac{K}{M}}$$

Its characteristic equation is

$$s^2 + \frac{B}{M}s + \frac{K}{M} = 0.$$

For a second order system the characteristic equation is a quadratic. The solution to the quadratic equation (its roots) has 6 forms. Each corresponds to a different second order response.

The form of the characteristic's equation's roots specifies the system's response!

Table 1 lists the 6 possible forms for the roots of a second order system's characteristic equation and the type of response which corresponds to each. Also listed is value for the **damping ratio**. The damping ratio is a measure of energy dissipation vs. energy storage in the system. The value of the damping ratio also corresponds to the type of response.

Roots of Characteristic Equation	Example	Response	Damping Ratio (ζ)
Real, Negative and Distinct	$s = -2$ and -4	Over Damped	$\zeta > 1$
Real, Negative and Repeated	$s = -2$ and -2	Critically Damped	$\zeta = 1$
Complex w/ Negative Real Parts	$s = -2 \pm 3i$	Under Damped	$0 < \zeta < 1$
Complex w/ NO real parts	$s = \pm 4i$	Marginally Stable	$\zeta = 0$
Complex w/ Positive Real Parts	$s = 2 \pm 3i$	Unstable Oscillating	$-1 < \zeta < 0$
Real and Positive	$s = 2$ and 4	Unstable Non-Oscillating	$\zeta \leq -1$

Table 1: The system response and range of ζ for the range of roots of s .

There is a pattern between the form of the roots and the system's response.

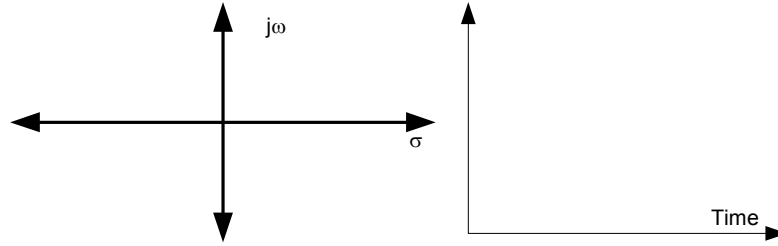
- If the roots are **complex**, the system **oscillates**.
- If **all** the roots' real parts are **negative**, the system is **stable**.
- If **any** of the roots' real parts are **positive**, the system is **unstable**. Zero is considered a positive number.

Lesson 3b: Second Order Systems' Time Response

1c) Plot the roots of the characteristic equation which would give the response. Sketch the step response in the time domain. Give the value for the damping ratio.

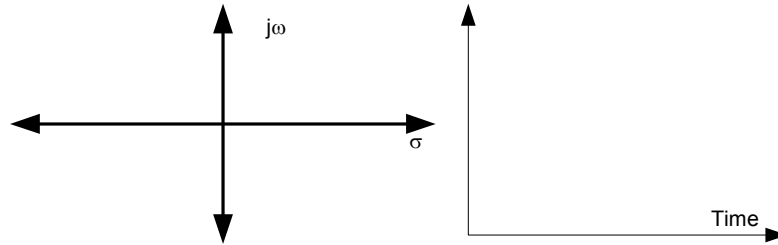
Over Damped

ζ :



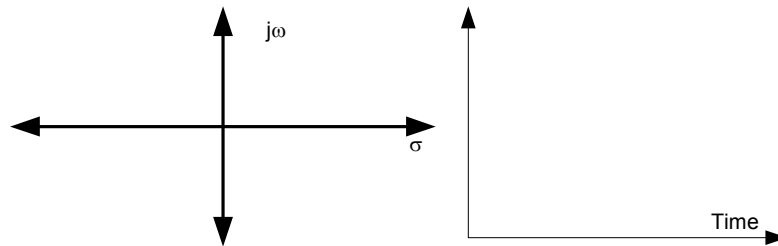
Critically Damped

ζ :



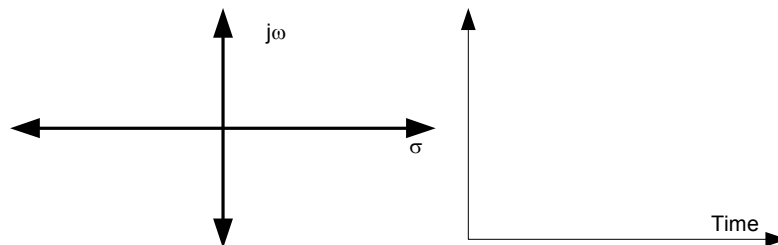
Under Damped

ζ :



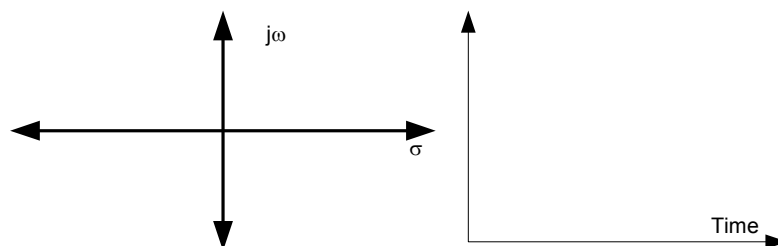
Marginally Stable

ζ :



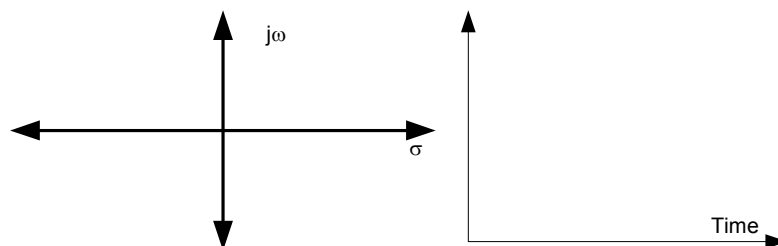
Unstable Oscillating

ζ :



Unstable Non-Oscillating

ζ :



Mass Spring Damper Example

Consider the following simple mass spring damper system. With the following properties: $M=10$ kg, $K=40$ N/m, and $B=20$ Ns/m

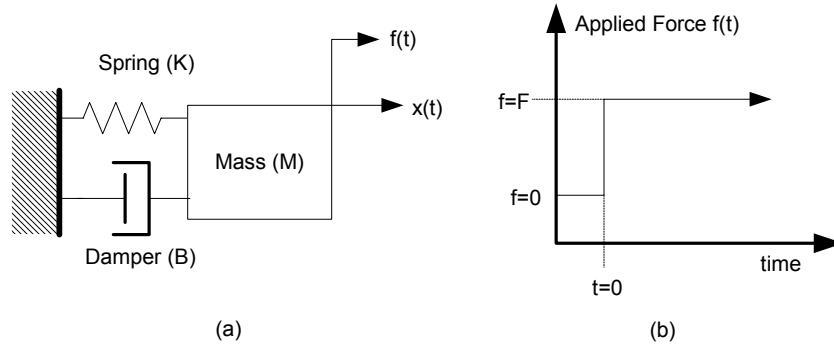


Figure 2: The mass spring damper system (a) and the force applied to it as a function of time (b).

Figure 2a illustrates the mass spring damper system. From time zero onwards a constant force F is applied to the system as illustrated by Figure 2b. The system's response (its motion) is measured by $x(t)$.

2a) What kind of wave form is this force input (Lesson 1c)?

2b) With an applied force F of 100N, how far do you expect the mass to move (Hook's Law)?

2c) Describe how the mass would move with no friction in the system (no damping). Make a sketch of $x(t)$.



2d) Describe how the mass would move with a small amount of friction in the system. Make a sketch of $x(t)$.



2e) Describe how the mass would move with a very large amount of friction in the system. Make a sketch of $x(t)$.



Lesson 3b: Second Order Systems' Time Response

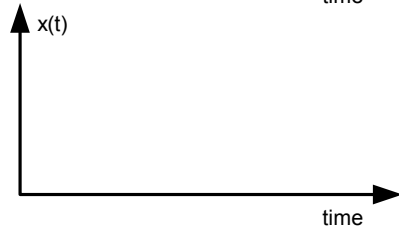
3a) Describe how the mass would move with no spring in the system. Make a sketch of $x(t)$.



3b) Describe how the mass would move with a weak spring in the system. Make a sketch of $x(t)$.



3c) Describe how the mass would move with a very stiff spring in the system. Make a sketch of $x(t)$.



The transfer function for this mass spring damper system is

$$G(s) = \frac{X(s)}{F(s)} = \frac{\frac{1}{M}}{s^2 + \frac{B}{M}s + \frac{K}{M}}$$

Input
Parameter

4a) For this transfer function which parameter is the input and which parameter is the output (Lesson 2b)?

Output
Parameter

4b) Use the final value theorem (Lesson 3a) to calculate the final position of the mass for the 100N step input. Use the above transfer function with the correct values of M, B, and K substituted. Does this answer agree with your answer to question 2b?

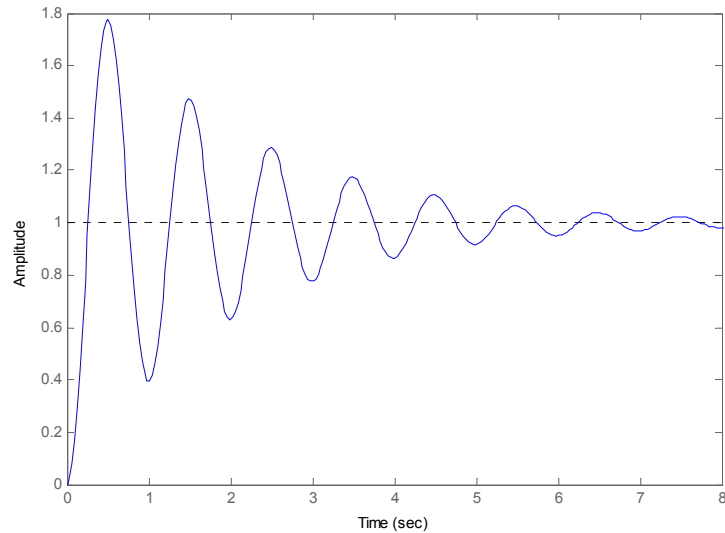
$x_{\text{final}} =$

4c) Which of these cases could the mass spring damper system exhibit. Assume the values for M, B and K are all greater than zero.

4d) Which of these cases could NOT be exhibited by the mass spring damper system? Why not? Assume the values for M, B and K are all greater than zero.

Lesson 3b: Second Order Systems' Time Response

5a) As the under damped mass spring damper system oscillates back and forth, its energy is stored in the different energy storage elements (the spring and the mass). Annotate the plot below with the points where the system's energy is stored entirely in the spring and where it is stored entirely in the mass.



5b) What is happening to the total energy in the system? How does the above plot show this? What is causing this?

Lesson 3b: Second Order Systems' Time Response

6) For each of the following transfer functions, find the characteristic equations, its roots, and its response type. J and K are tricky!

	Transfer Function	Characteristic Equation	Roots	Response Type Name & Sketch
a	$\frac{2}{s^2 + 5s + 6}$			
b	$\frac{4}{2s^2 - 8s + 4}$			
c	$\frac{20}{2s^2 + 6s + 10}$			
d	$\frac{16}{s^2 + 16}$			
e	$\frac{100}{s^2 + 10s + 25}$			
f	$\frac{8}{s^2 - 2s + 8}$			
g	$\frac{4.25}{10s^2 + 12s + 3.6}$			
h	$\frac{10}{6s^2 + 3s + 10}$			
i	$\frac{22}{s^2 + 8s + 11}$			
j	$\frac{2}{s^2 + 5s}$			
k	$\frac{2}{s^2 + 8s - 2}$			

Using ltiview

Next we will use MATLAB to test our predictions for the mass spring damper's response. See the **MATLAB help section of the workbook to learn how to enter transfer functions in MATLAB.**

Write the system's transfer function, substituting the values of M, B, and K into the transfer function.

G(s) =

7a) To display the system's response to a **unit** step input, enter the transfer function into MATLAB and run the command `ltiview('step',G)`.¹ What is the final displacement of the mass as displayed by MATLAB? Does this match your predictions in questions 2b and 10? Why or why not?

$x_{\text{final}} =$

7b) Using `ltiview` answer questions 6 through 11 again.

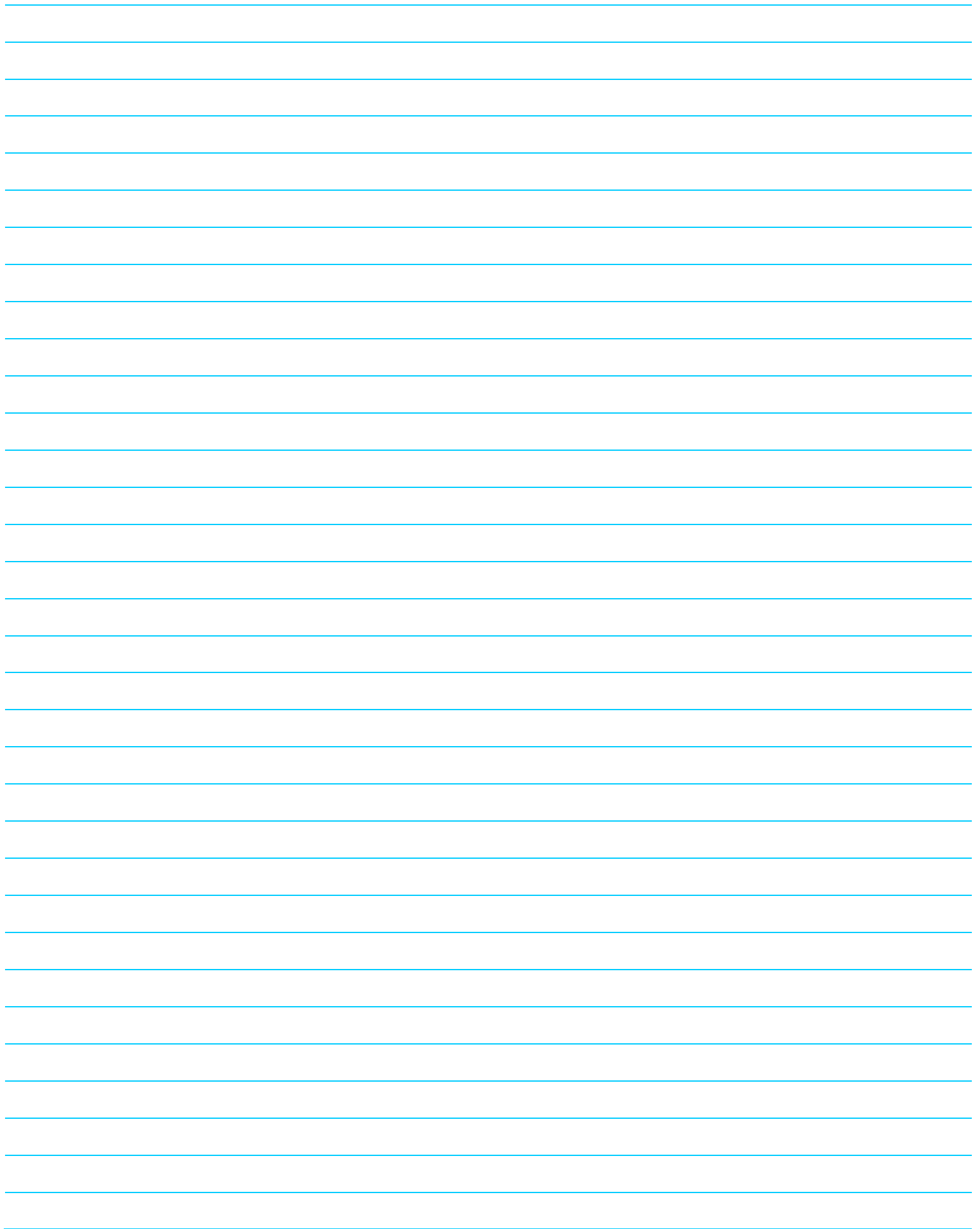
- Print two plots. One plot with questions 2c, 2d, and 2e on the same graph, and another plot with questions 3a, 3b, and 3c on one graph.
- Label the curves with their question number and “high friction”, “low friction”, “no friction”, “strong spring”, “weak spring”, or “no spring” as appropriate.

Write the transfer function used for each curve below.

G _{2c} (s) =	G _{3a} (s) =
G _{2d} (s) =	G _{3b} (s) =
G _{2e} (s) =	G _{3c} (s) =

20) Which aspects of the time response were affected by changing the friction (B) and spring stiffness (K)?

¹ Assuming the transfer function was saved in MATLAB as the variable “G”. If the variable was named something else, then use that name instead of “G”.



Time Response Parameters

A system's time response is measured by several quantitative parameters calculated from a system's characteristic equation. The general form of the characteristic equation is

$$s^2 + (2\zeta\omega_n)s + \omega_n^2 = 0.$$

In the general form, the coefficient of the s^2 term must be 1!

Parameter	Description
Sigma σ	Sigma is found from the roots of the characteristic equation. It is the opposite of the root's real part. For example, if the roots are $s = -2 \pm 4i$, then $\sigma = 2$. For critically and under damped systems, $\sigma = \zeta\omega_n$. For over damped systems where the roots are real and distinct, there are two sigma values. For example, if the roots are $s = -4 \& -10$, then $\sigma_1 = 4$ and $\sigma_2 = 10$.
Damped Frequency of Oscillation ω_d	This is the frequency the system oscillates at. ω_d is the imaginary part of the characteristic equation's roots. For example, if the roots are $s = -2 \pm 4i$, then $\omega_d = 4 \frac{rad}{sec}$. Remember $f_{Hz} = \frac{1}{T}$ and $f_{\frac{rad}{sec}} = \frac{2\pi}{T}$. ω_d must be in $\frac{rad}{sec}$ for calculations!
Damping Ratio ζ	This is a measure of how much friction or energy dissipation there is in a system. $\zeta = 0$ corresponds to no friction. $\zeta = 1$ corresponds to just enough friction to keep the system from oscillating back and forth (critically damped). If $\zeta > 1$ the system is over damped.
Natural Frequency ω_n	ω_n combines the effects of both ζ and ω_d . This is the frequency the system will oscillate at with no friction in the system.
Settling Time $T_s = \frac{4}{\sigma}$	The time at which the response no longer exceeds 2% of its final value. For under and critically damped systems $T_s = \frac{4}{\sigma} = \frac{4}{\zeta\omega_n}$. For over damped systems, use the sigma value which makes the settling time longest!
Peak Time $T_p = \frac{\pi}{\omega_d}$	The time at which the function reaches its maximum peak. For critically and over damped systems, $\omega_d = 0$ (no imaginary part) therefore the peak time is infinite (it never has a peak).
Percent Overshoot $\%OS = 100e^{-\frac{4T_p}{T_s}}$	The amount the function's maximum peak exceeds the final value, as a percentage of the final value. $\%OS = 0$ for critically and over damped systems, $\omega_d = 0 \Rightarrow T_p = \infty$ and $e^{-\infty} = 0$.

The Three Questions

When analyzing a second order system there are three important questions.

1. **Is the system stable?** Use the roots of the characteristic equation to determine stability.
2. **Is the system's steady state value acceptable?** Alternatively, how much is the error from the desired value? Use the final value theorem to determine if the system's final value is adequate for the design or application.
3. **Is the system's transient response acceptable?** There are several aspects of the system's transient response which could make the response unacceptable:
 - a. Does the system get to its final value fast enough (settling time)?
 - b. Does the system overshoot its final value too much (%OS)?
 - c. Is the number of oscillations too many (ζ)?
 - d. Is the frequency of oscillation too high (ω_d)?

Measuring the Time Response Parameters

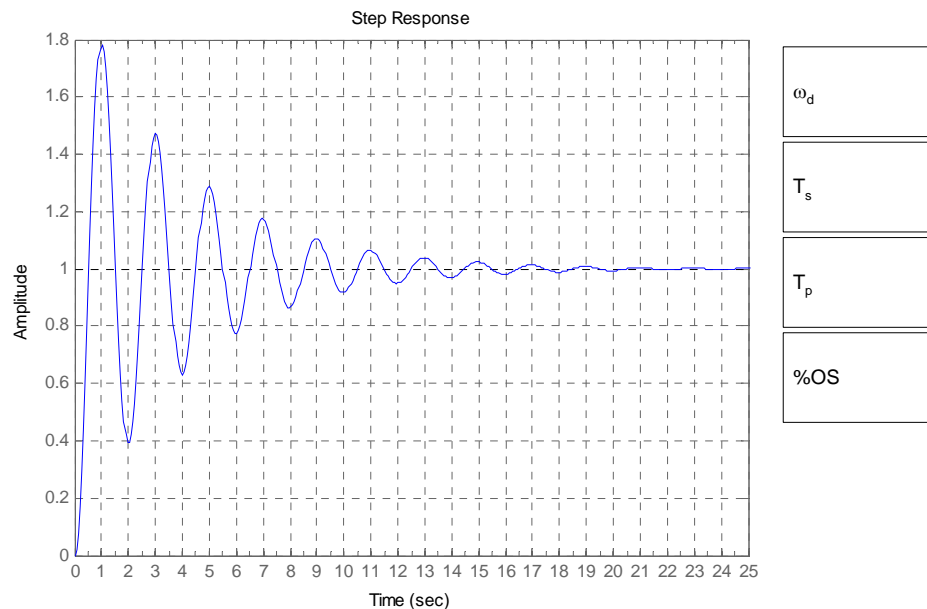


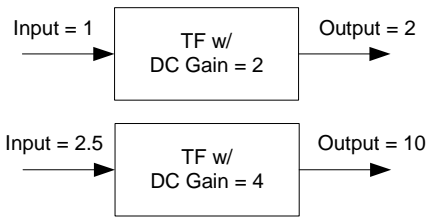
Figure 1: The time response of an under damped system.

1) Using Figure 1, find the values for ω_d , T_s , T_p , and %OS. Annotate the plot to show where the values were read from.

Lesson 4: Second Order Time Response Calculations

DC Gain

The DC gain of a transfer function is the amount it amplifies (multiplies) the input by when all the transients have died out, as time goes to infinity. Time going to infinity is the same as frequency going to zero ($f = 1/T$).



The DC Gain is found by taking the limit of the transfer function as s approaches 0. The DC Gain can also be found when system's input and its steady state output are known.

2) Complete the table below.

	Transfer Function or Response	DC Gain
a	$G(s) = \frac{5}{s^2 + 3s + 5}$	
b	$G(s) = \frac{10}{s^2 + 3s + 1}$	
c	$G(s) = \frac{6}{10s^2 + 2s + 3}$	
d	<p>Input: Unit Step</p>	
e	<p>Input: Step of Magnitude 3</p>	

Calculating Time Response Parameters

The system's time response parameters can also be calculated directly from the characteristic equation for the transfer function.

Consider the transfer function

$$G(s) = \frac{10}{s^2 + 0.5s + 10}.$$

3a) Write the transfer function's characteristic equation.

Characteristic Eqn

If the coefficient of the s^2 term is one, then the terms of the characteristic equation map nicely to several time response parameters. The general form of the characteristic equation is

$$s^2 + (2\zeta\omega_n)s + \omega_n^2 = 0.$$

Additionally, the roots of the characteristic equation also map to several time response parameters. The general forms for the roots of the characteristic equation are

$$s = -\sigma \pm \omega_d i = -\zeta\omega_n \pm \omega_d i.$$

3b) Find the roots of the characteristic equation from question 3a and label the horizontal and vertical axes of Figure 2 with the correct symbolic values.

Roots of Characteristic Eqn

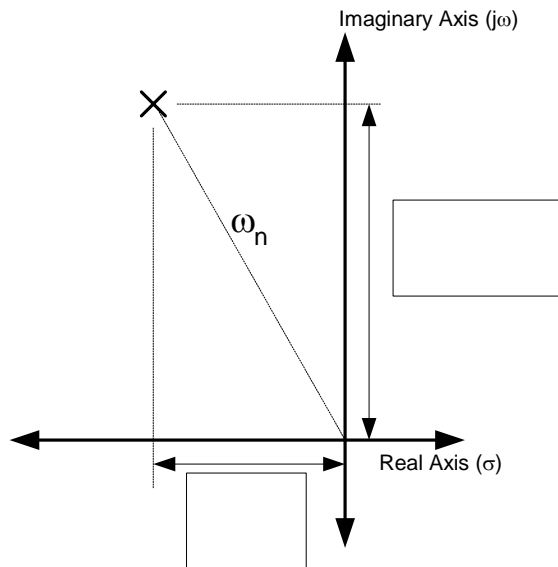


Figure 2: A root of the characteristic equation plotted on the complex plane. This shows the relationship between ω_n , ω_d , and ζ .

Lesson 4: Second Order Time Response Calculations

3c) Write an equation which describes the relationship between ω_n , ω_d , and ζ .

Eqn

3d) Calculate ω_n , ω_d , and ζ for the characteristic equation in question 3a. Calculating $\zeta\omega_n$ is often a useful intermediate step.

ω_n	$\zeta\omega_n$	ω_d	ζ
------------	-----------------	------------	---------

After the parameters ω_n , ω_d , and ζ have been calculated, the remaining time response parameters are found with the equations:

$$T_s = \frac{4}{\sigma}, T_p = \frac{\pi}{\omega_d}, \text{ and } \%OS = 100e^{\frac{-4T_p}{T_s}}.$$

Remember, when calculating settling times for over damped systems, use the σ which gives the longest settling time!

3e) Find the settling time, peak time, and percent overshoot for this system.

T_s	T_p	%OS
-------	-------	-----

3f) Complete the table below (pay attention to j).

	Characteristic Equation	Roots	Response Type	ω_n	$\zeta\omega_n$	ζ	ω_d	Ts	Tp	%OS
a	$s^2 + 4s + 8 = 0$									
b	$s^2 + 7s + 23 = 0$									
c	$s^2 + 50s + 10 = 0$									
d	$s^2 + s + 1 = 0$									
e	$s^2 + 3s + 9 = 0$									
f	$s^2 + 36s + 6 = 0$									
g	$s^2 + 9s + 30 = 0$									
h	$2s^2 + 14s + 46 = 0$									
i	$s^2 + 9s + 20 = 0$									
j	$s^2 + 12s + 36 = 0$									

Lesson 4: Second Order Time Response Calculations

4a) Using a sketch of the complex plane, explain why the natural frequency is the frequency the system oscillates at when there is no friction.

Hint: What is the value of ζ when there is no friction.

4b) For the transfer function $G(s)$ explain how **increasing** b , c , and d will affect the time response parameters.

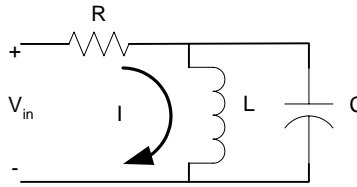
Hint: A sketch of the complex plane will be useful.

$$G(s) = \frac{d}{s^2 + b \cdot s + c}$$

Parameter	ω_n	$\zeta\omega_n$	ζ	ω_d	T_s	T_p	%OS	DC Gain
$b \uparrow$								
$c \uparrow$								
$d \uparrow$								

Lesson 4: Second Order Time Response Calculations

5) Answer the following questions about the RLC circuit pictured below.



a) Write a transfer function for this system with voltage as the input and current as the output.

b) If a step input of 10 volts is applied to system what is the steady state current? Explain physically why this happens.

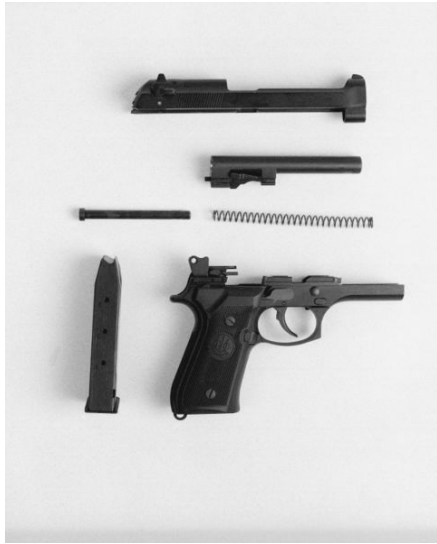
c) If the resistor has a value of 6 Ohms and the Inductor is 1mH, how big should the capacitor be to make the system critically damped?

d) As the value of the resistor **increases** explain what happens to each of the time response parameters.

Parameter	ω_n	$\zeta\omega_n$	ζ	ω_d	T_s	T_p	%OS	DC Gain
$R \uparrow$								

Lesson 4: Second Order Time Response Calculations

6) Answer the following questions about the recoil mechanism of the Beretta M9 Pistol.



a) Write a transfer function for this system describing the recoil mechanism's position for an applied force.

b) Label the components in the photos above which contribute to each parameter in the transfer function.

d) As the stiffness of the spring **decreases** explain what happens to each of the time response parameters.

Parameter	ω_n	$\zeta\omega_n$	ζ	ω_d	T_s	T_p	%OS	DC Gain

Graphical Analysis of Second Order Responses

Transfer functions can be estimated for a system without knowing the internal configuration of the system. All that is required is to measure the system's output as a response to a known input. With this information it is possible to solve for the system's transfer function. This is a powerful and useful concept, particularly when modeling actual hardware.

First Order Review

The concept of deriving a transfer function from a response was introduced for first order systems in Lesson 2b. Recall that the transfer function for a first order system is

$$G(s) = \frac{a}{s + \sigma}.$$

Three steps are required to find a first order transfer from a system's step response.

1. Solve for σ by **either**:

a) Find the time the response is at 63% of its final value. This time is the time constant $\frac{1}{\sigma}$. Solve for σ .

or

b) Find the settling time for the system (the response is at 98% of its steady state value¹). Use $T_s = \frac{4}{\sigma}$ to solve for σ .

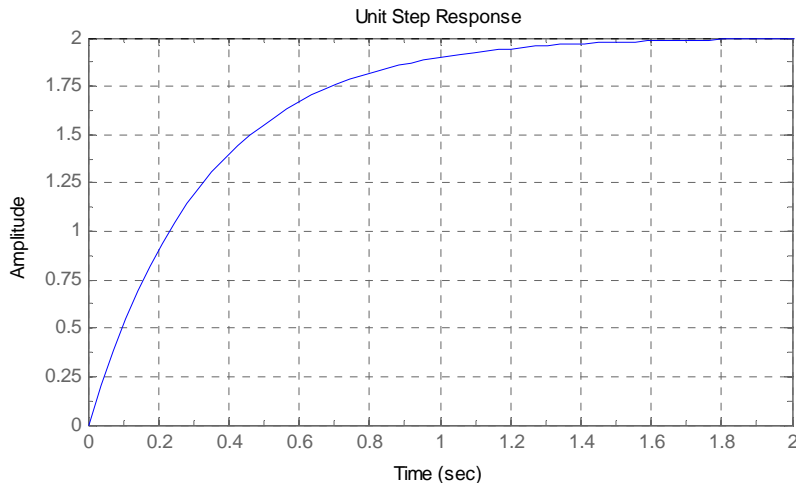
Method 1a is usually a more accurate way of determining σ .

2. Divide the system's steady state value by the magnitude of the step input to find its DC gain. A unit step has a magnitude of 1.
3. The DC gain is the value of the transfer function as s approaches zero. Use the DC gain and σ to solve for the numerator of the transfer function.

$$DC_{gain} = \lim_{s \rightarrow 0} \frac{a}{s + \sigma} = \frac{a}{\sigma} \Rightarrow a = DC_{gain} \sigma$$

¹ The system is considered to be at steady state when it is within 2% of its final value (98% of the system's steady state value).

1) Find the first order transfer function for the following plot:



G(s)=

Second Order Systems

Second order systems are more complicated because their transfer functions have more variables and the system can exhibit several possible responses.

The general forms for a second order transfer function and its characteristic equation are:

$$G(s) = \frac{d}{s^2 + b \cdot s + c} \text{ and } s^2 + b \cdot s + c = s^2 + 2\zeta\omega_n \cdot s + \omega_n^2 = 0.$$

There are 6 major steps to finding the transfer function for a second order response.

1. Measure the Peak Time, T_p , and calculate the damped frequency of oscillation, ω_d .
2. Measure the system's steady state value, x_{ss} , and peak value, x_{peak} .

3. Calculate %OS with $\%OS = \frac{x_{peak} - x_{ss}}{x_{ss}}$.

4. Using the peak time, T_p , and %OS solve for $2\zeta\omega_n$. $\%OS = e^{\frac{-4T_p}{T_s}} = e^{-T_p \zeta \omega_n}$ yields

$$\zeta\omega_n = \frac{-\ln(\%OS)}{T_p}.$$

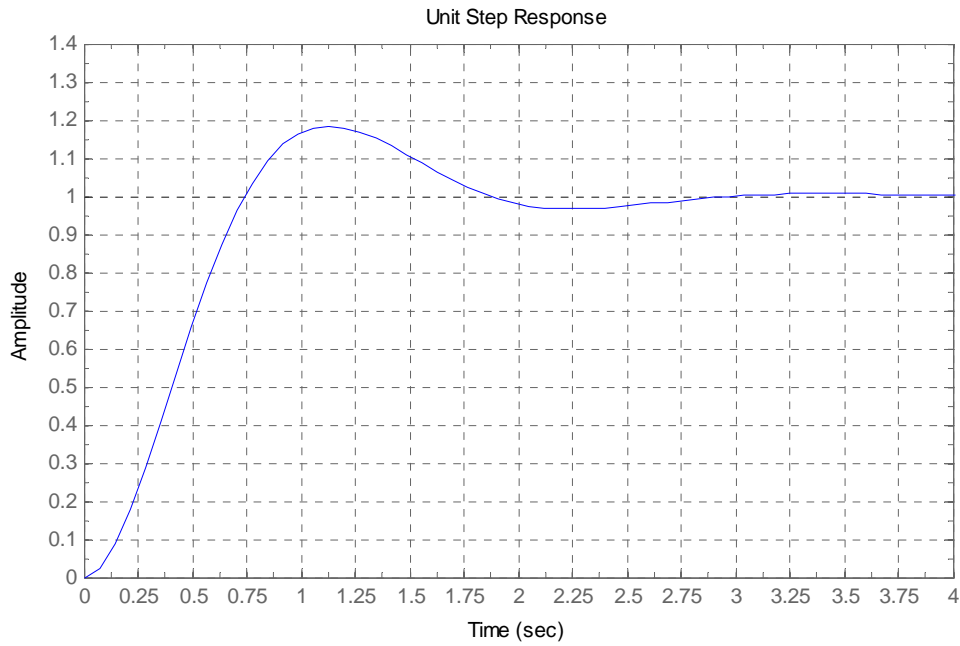
5. Solve for the natural frequency, ω_n , with $\omega_n = \sqrt{\omega_d^2 + (\zeta\omega_n)^2}$.

6. Solve for the coefficients of the transfer function using

$$b = 2\zeta\omega_n, \quad c = \omega_n^2, \quad \text{and } d = \frac{c \cdot x_{ss}}{\text{magnitude}_{input}}.$$

Lesson 5a: Graphical Analysis of Second Order Responses

2) A unit step applied to a system generates the following response. Use the plot to find the values for: ω_d , T_s , T_p , %OS, and DC Gain.

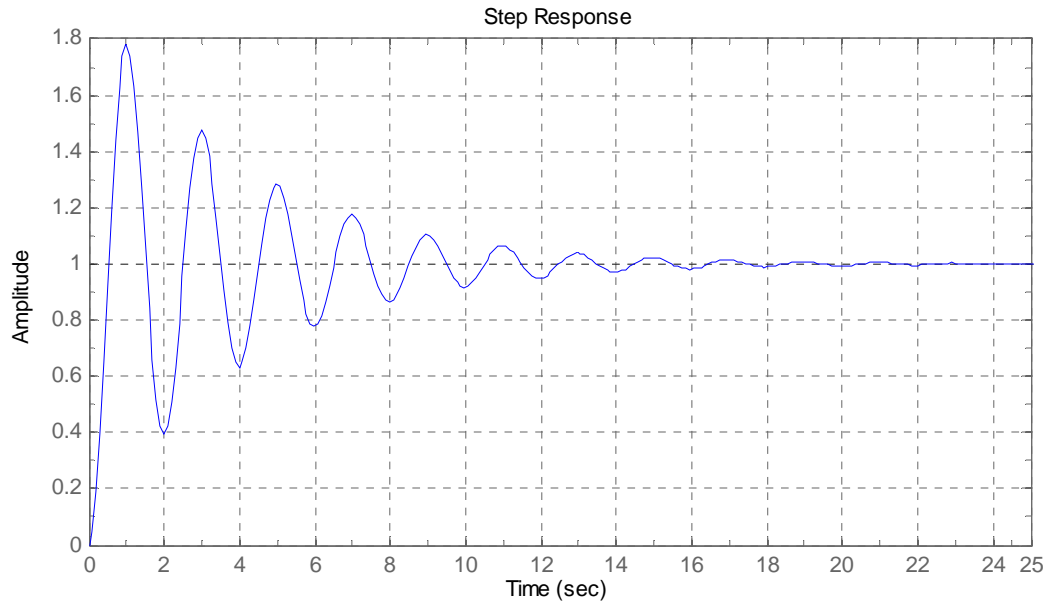


$\omega_d =$ _____ $T_s =$ _____ $T_p =$ _____ %OS = _____ DC Gain = _____

Find the transfer function which gives the response depicted above.

Lesson 5a: Graphical Analysis of Second Order Responses

3) A step of **magnitude 2** is applied to a system generates the following response. Use the plot to find the values for: ϖ_d , T_s , T_p , %OS, and DC Gain.

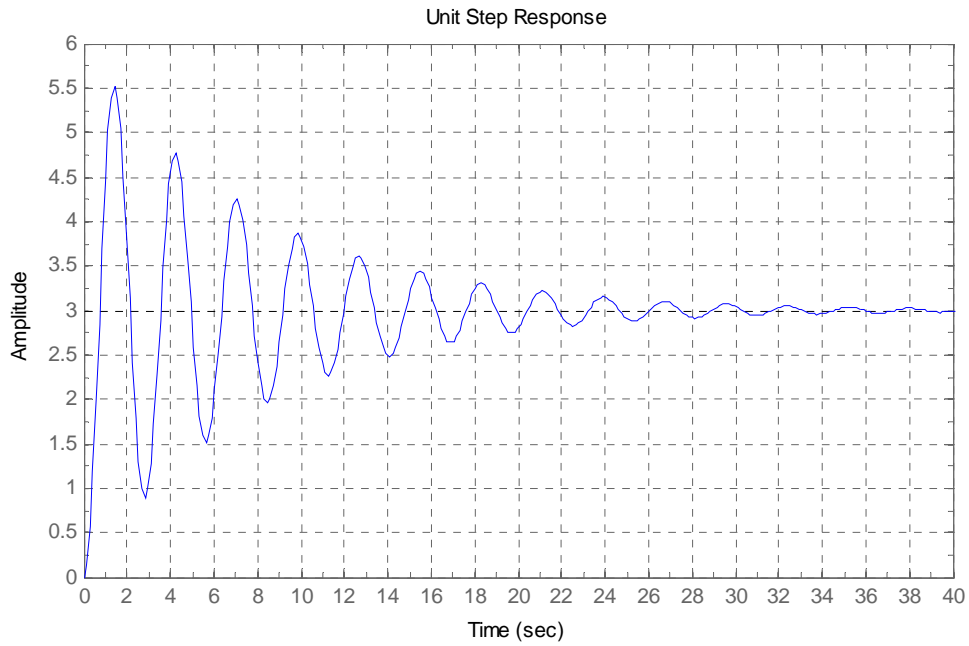


$\varpi_d =$ _____ $T_s =$ _____ $T_p =$ _____ %OS = _____ DC Gain = _____

Find the transfer function which gives the response depicted above.

Lesson 5a: Graphical Analysis of Second Order Responses

4) A unit step applied to a system generates the following response. Use the plot to find the values for: ω_d , T_s , T_p , %OS, and DC Gain.

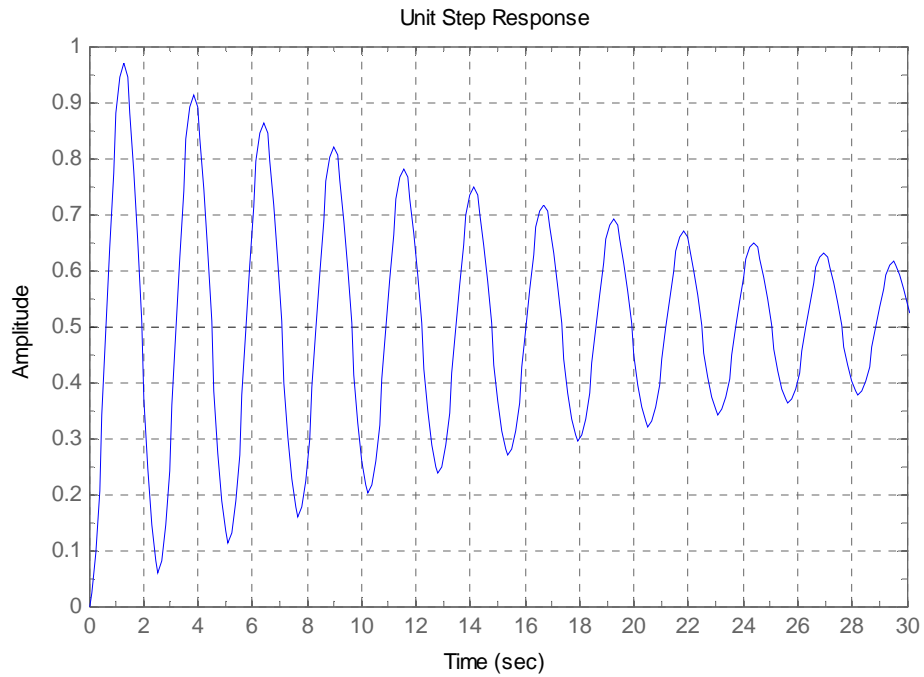


$\omega_d =$ _____ $T_s =$ _____ $T_p =$ _____ %OS = _____ DC Gain = _____

Find the transfer function which gives the response depicted above.

Lesson 5a: Graphical Analysis of Second Order Responses

5) A unit step applied to a system generates the following response. Use the plot to find the values for: ω_d , T_s , T_p , %OS, and DC Gain.

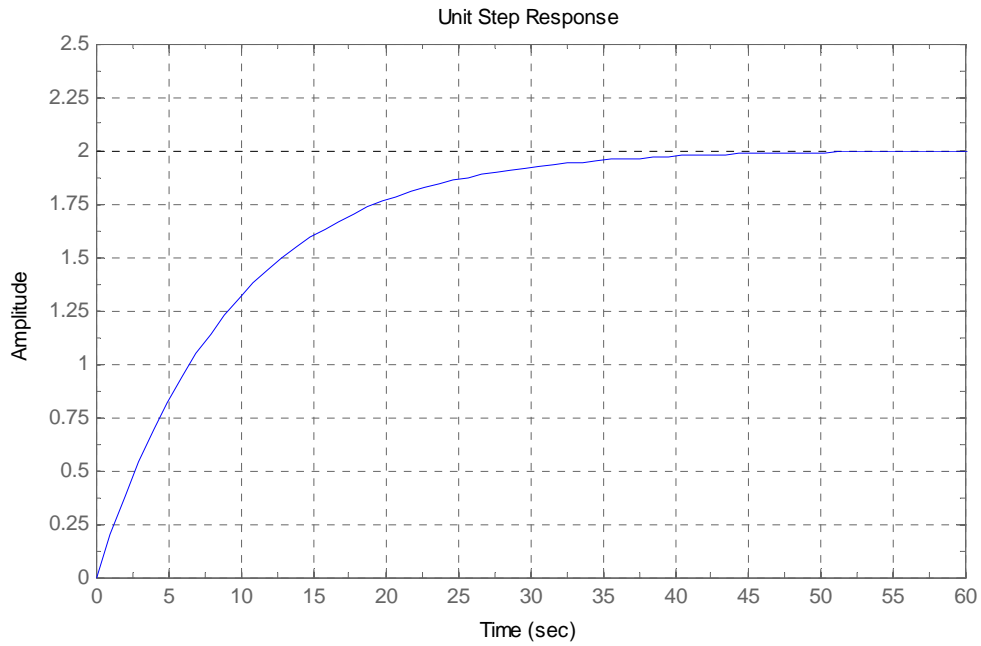


$\omega_d =$ _____ $T_s =$ _____ $T_p =$ _____ %OS = _____ DC Gain = _____

Find the transfer function which gives the response depicted above.

Lesson 5a: Graphical Analysis of Second Order Responses

6) A unit step applied to a system generates the following response. Use the plot to find the values for: ω_d , T_s , T_p , %OS, and DC Gain.

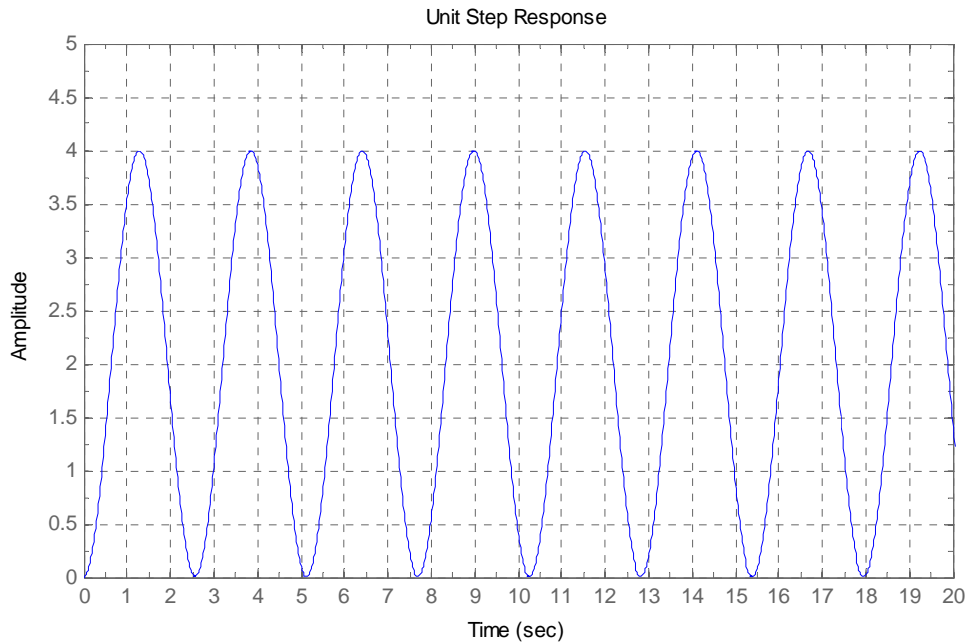


$\omega_d =$ _____ $T_s =$ _____ $T_p =$ _____ %OS = _____ DC Gain = _____

Find the transfer function which gives the response depicted above. **Hint: There is an easy and a difficult way of solving this question.**

Lesson 5a: Graphical Analysis of Second Order Responses

7) A unit step applied to a system generates the following response. Use the plot to find the values for: ω_d , T_s , T_p , %OS, and DC Gain.



$\omega_d =$ _____ $T_s =$ _____ $T_p =$ _____ %OS = _____ DC Gain = _____

Find the second order transfer function which gives the response depicted above.

Compare the values of ω_d and ω_n . What is significant about the results of this comparison? Using the complex plane explain why.

Measurement of an Actual Second Order Response

This lesson will examine the step response of a second order mass spring damper system. The time response will be used to determine the transfer function of the system.

Lesson 5a, Analysis of Second Order Responses, must be completed before starting this lesson.

The System

The system consists of a vertical mass, spring, and damper. Two variants of a vertical mass spring damper system are shown in Figure 1.

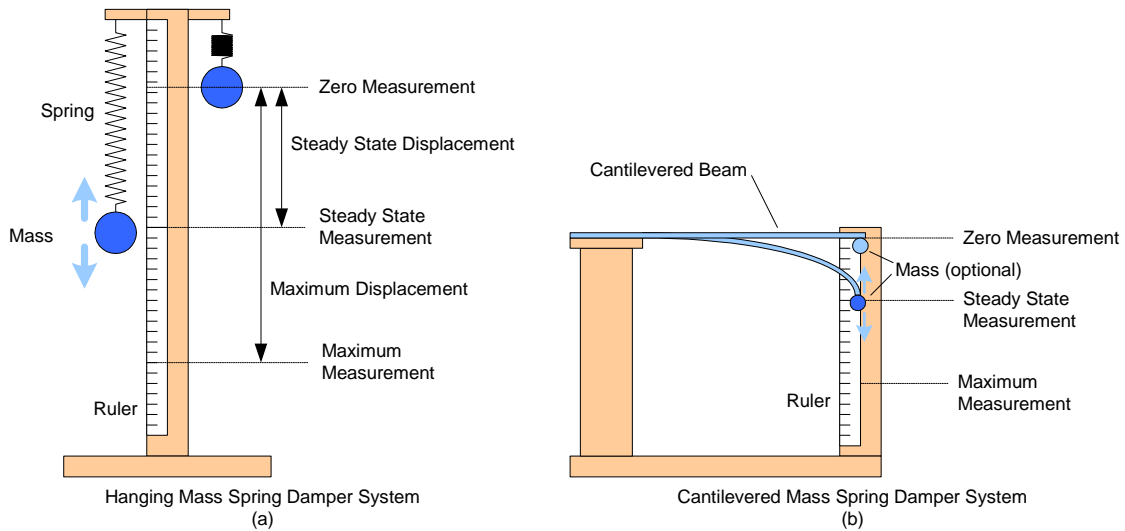


Figure 1: A hanging mass spring damper system with separate components (a).
A cantilevered beam is also a vertical mass spring damper system (b).

This system is slightly different from mass spring damper systems presented in Lessons 3 and 4. Because it is vertical, the applied force is proportional to the system's mass

$$F = mg .$$

At time zero the mass will be released and the gravitational force applied as a step input to the system.

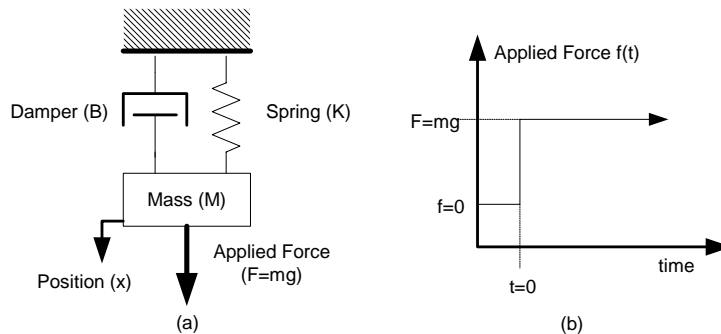


Figure 2: The model of the mass spring damper system (a) and the gravitational force applied as a step input (b).

Lesson 5b: Measurement of an Actual Second Order Response

1) Write a second order transfer function which relates the position of the mass to the applied force.

G(s) =

2a) Of the three parameters in the transfer function, which is the easiest to measure? Which is the most difficult to measure?

Easiest to Measure	Hardest to Measure
---------------------------	---------------------------

2b) The damper component is not labeled in Figures 1a and 1b. Where is the damper in these systems?

2c) Which parameters of the transfer function change as the cantilevered beam is shortened? How do these parameters change?

3) Using the final value theorem, solve for the steady state position of the mass in term of the parameters in the transfer function.

X_{ss} =

4) Write the six steps for deriving a second order transfer function from the under damped response of a system.

Lesson 5b: Measurement of an Actual Second Order Response

This lesson can be accomplished with either the hanging mass spring damper shown in Figure 1a or the cantilevered beam shown in Figure 1b.

All calculations should be done in metric units¹.

5) Measure or calculate the following:

Lab Setup (Circle One):		Hanging Mass		Cantilevered Beam	
Line	Parameter	English	Metric		Notes
1	Gravity	g	m/s ²	9.8	
2	Mass	M	kg		
3	Zero Measurement	in	m		
4	Steady State Measurement	in	m		
5	Steady State Displacement	X _{ss}	in	m	line 4 - line 3
6	Spring Constant	K	N/m		
7	Cantilevered Beam Length	L	m		Cant. Beam Only

Release the mass from the zero measurement point and record the peak time, period of oscillation, and maximum displacement. Perform four trials and average the results.

It is easiest to have a single person dedicated to measuring each parameter (Tp, period of oscillation, and maximum measurement).

Trial	Period of Oscillation	Peak Time	Maximum Measurement
1			
2			
3			
4			
Average			

6a) From the average maximum **measurement** calculate the average maximum **displacement**, see Figure 1. Calculate the %OS for the system.

Average Maximum Displacement	Average %OS
------------------------------	-------------

¹ 1 inch = 2.54 cm = 0.0254 m 1 lb = 0.453 kg = 453 grams

Lesson 5b: Measurement of an Actual Second Order Response

6b) Use the period of oscillation and the peak time data to calculate two values for the damped frequency of oscillation, ω_d .

The ω_d calculated from the peak time is already in rad/sec. The ω_d calculated from the measured period of oscillation must be converted from 1/sec to rad/sec. (see Lesson 4)

$\omega_d =$ From Period Measurement	$\omega_d =$ From Peak Time Measurement
---	--

6c) Which value of ω_d do you have more confidence in? Why?

6d) Solve for the transfer function of the system and find the value of the friction term B.

G(s) =

B =

7) For the step input of **magnitude F=mg** applied to the above transfer function, calculate the time response parameters.

Characteristic Equation			% Diff from Measured
Roots of the Characteristic Equation		ω_d	
System Response Type: (Over Damped, Critically Damped, Under Damped, Unstable)		Ts	
ω_n		Tp	
$\zeta\omega_n$		%OS	
ζ		DC Gain	

Calculate the percent difference between the measured values and calculated values for ω_d , Ts, Tp, %OS, and DC gain.

$$\% \text{ diff} = 100 \left(\frac{\text{calculated} - \text{measured}}{\text{measured}} \right)$$

Modeling an F/A-18 Landing

The landing gear on an aircraft is designed to support the aircraft and dissipate its energy during landing. If the landing gear is compressed completely on landing, any remaining energy will be dissipated by the airframe possibly damaging it.

For this simulation the aircraft and its landing gear are modeled as the “spring mass damper” system illustrated in Figure 1.

The weight of the aircraft changes during flight as fuel is burned and ordnance is released. Typical limits and values for an F/A-18 are listed in Table 1.



Figure 1: The spring mass damper model of an F/A-18 landing gear.

F/A-18 Model Parameters			
Basic Empty Weight	M_{BEW}	14,300	Kg
Maximum Landing Weight, Carrier	M_{MLC}	19,900	Kg
Maximum Landing Weight, Field	M_{MLF}	22,700	Kg
Maximum Take Off Weight	M_{MTO}	29,900	Kg
Damper	B	150,000	Ns/m
Spring	K	700,000	N/m
Maximum Landing Gear Compression	y_{max}	1	m

Table 1: The F/A-18 model parameters and typical loading conditions.

Questions:

1) If an F/A-18 at maximum takeoff weight and has an emergency shortly after take off, how much fuel and ordnance must be jettisoned prior to landing at an airfield? How much would have to be jettisoned for landing on a carrier? Report your answers in both kg and tons (1kg = 2.2 lbs and 1 ton = 2000 lbs).

	kg	tons
Weight jettisoned for landing at an airfield		
Weight jettisoned for landing on a carrier		

ES360 Introduction to Controls Engineering
Lesson 5c: Modeling an F/A-18 Landing

2) Calculate the expected steady state displacement of the aircraft's landing gear, in meters, for all four of the loading conditions (use Hook's Law).

Found Via Hooks Law	M_{BEW}	M_{MLC}	M_{MLF}	M_{MTO}
Steady State Gear Displacement [m]				

Use MATLAB and the Simulink model of the F/A-18's landing gear to answer questions 3 through 5. The section "Running the F/A-18 Simulation" on page 4 has step by step instructions for using this Simulink model.

3) Check your answers to question 2. How closely did the simulation match your answers?

Found via Simulink	M_{BEW}	M_{MLC}	M_{MLF}	M_{MTO}
Steady State Gear Displacement [m]				

4) Find the maximum decent rate, in feet per minute [fpm] and maximum acceleration [G's], for **field** landings which will not fully compress the landing gear.

Max Decent Rate, Carrier Landing [fpm]

Max Acceleration, Carrier Landing [G's]

5) Find the maximum decent rate, in feet per minute [fpm] and maximum acceleration [G's], for **carrier** landings which will not fully compress the landing gear.

Max Decent Rate, Field Landing [fpm]

Max Acceleration, Field Landing [G's]

6) Explain why changing the mass of the aircraft changes the maximum allowable decent rate and maximum acceleration of the aircraft.

Lesson 5c: Modeling an F/A-18 Landing

7) A more typical decent rate on landing is -600 fpm. Find the maximum acceleration and minimum distances from full landing gear compression for field and carrier landings

Landing at -600 fpm & Max Landing Weight	Maximum Gear Compression [m]	Maximum Acceleration [G's]
Field		
Carrier		

8) Run the simulation **and print** the results, with a decent rate of -600 fpm at maximum carrier landing weight. Run the simulation with a spring stiffness of 350,000 N/m (half stiffness) and 1,400,000 N/, (twice stiffness). **Print these results.**

As spring stiffness increases how does maximum displacement, steady state displacement, maximum acceleration, the damping ratio (ζ) change, settling time, and %OS change? Put your answers in the table below.

9) Set K back to its original value of 700,000 N/m. At a decent rate of -600 fpm and maximum carrier landing weight, run the simulation again with a damper coefficient of 75,000 Ns/m (half value) and 300,000 Ns/m (twice value). **Print these results.**

As the damper's value increases, how does maximum displacement, steady state displacement, maximum acceleration, the damping ratio (ζ) change, settling time, and %OS change? Put your answers in the table below.


Parameter	Change as K increases (↑, ↓ or ↔)	Change as B increases (↑, ↓ or ↔)
Max Gear Displacement		
Steady State Gear Displacement		
Maximum Acceleration		
Damping Ratio (ζ)		
Settling Time (T_s)		
%OS		

Running the F/A-18 Simulation:

- I. Ensure the files **f18landing.m** and **f18.mdl** are in a directory you have access too (desktop, thumb drive, etc.).
- II. Use the browse [...] button at the top of the MATLAB command window to set the current (working) directory to the location where the files from step I are.
- III. Enter the values for the model parameters into MATLAB

```
>> M=19900; B=150000; K=700000;
```
- IV. Enter a guess for the initial vertical velocity and enter this into MATLAB. Decent rates are entered as **negative** values in feet per minute.

```
>> Vo_fpm=-400;
```
- V. Run the f18landing command.

```
>> f18landing
```
- VI. This command uses the Simulink model to generate plots of landing gear deflection vs. time, aircraft vertical velocity vs. time, and aircraft vertical acceleration vs. time. The f18landing command uses the variables Vo_fpm, M, B, and K to run the simulation. To run a simulation with different parameters, update the value of the parameter in MATLAB and rerun f18landing.
- VII. Use the data cursor  to read exact values from the plots.

Introduction to Feedback

The concept of feedback is the central idea behind control systems. Feedback is the process by which a system's output is compared to its input and adjustments are made to achieve the desired results.

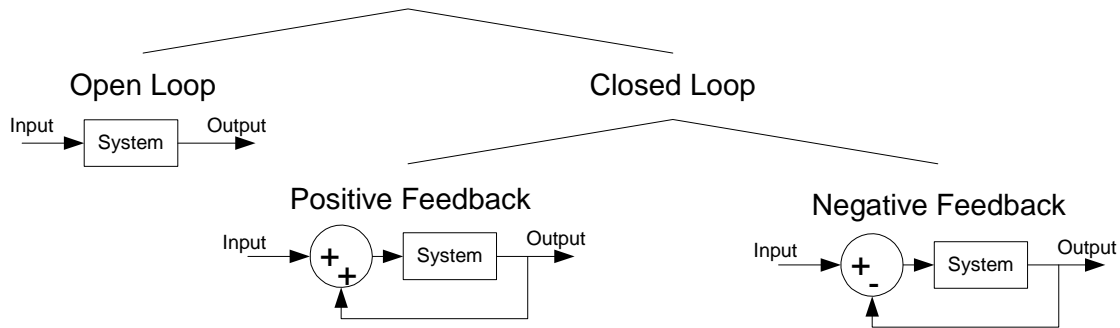


Figure 1: The relationship between open loop, closed loop, and positive/negative feedback and typical block diagrams for each.

Open Loop Control

“Open Loop” control is the case where there is NO feedback between the system's output and input. Examples of this include:

- Walking with one's eyes shut
- A single shot of an unguided projectile
- A microwave oven heating a meal

Closed Loop Control

“Closed Loop” control is the case where there IS feedback between the system's output and input. There are two possible ways to apply the feedback signal to the system. The feedback signal can be added to or subtracted from the system's input signal. These are referred to as positive or negative feedback. Note the sign difference between the positive and negative feedback loops' block diagrams in Figure 1.

Because negative feedback systems have a stabilizing effect on the output, they are much more common than positive feedback systems. A positive feedback system will drive its output to an extreme value.

An Automobile's Cruise Control Example

The Negative Feedback Case

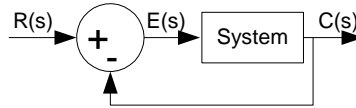


Figure 2: A simple negative feedback loop with the input $R(s)$, error $E(s)$, and output $C(s)$ signals labeled.

The cruise control in a car is an excellent example of a negative feedback system.

Write the equation for the error signal $E(s)$ for the negative feedback loop in Figure 2. Lesson 1b may be of some help.

$E(s)=$

If the error signal is positive, the control system opens the throttle more because the desired speed, $R(s)$, is greater than actual speed, $C(s)$. If the error signal is negative, the control system closes the throttle because desired speed, $R(s)$, is less than actual speed, $C(s)$. When the actual speed of the car is equal to the desired speed of the car, the error signal is zero and the control system does not change the throttle position.

If $R(s)$, the desired speed, is set to 55mph and the actual speed of the car, $C(s)$, is 45 mph, what is the value for $E(s)$? How does the cruise control change the throttle position?

$E(s)=$ at 45mph	Throttle Position (circle one) <input type="radio"/> Open More <input type="radio"/> Close More <input type="radio"/> Hold Position
---------------------	--

As the car speeds up to 50 mph what happens to the magnitude of $E(s)$? How does the cruise control change the throttle position?

$E(s)=$ at 50mph	Throttle Position (circle one) <input type="radio"/> Open More <input type="radio"/> Close More <input type="radio"/> Hold Position
---------------------	--

What is the value of $E(s)$ at 55mph? How does the cruise control change the throttle position?

$E(s)=$ at 55mph	Throttle Position (circle one) <input type="radio"/> Open More <input type="radio"/> Close More <input type="radio"/> Hold Position
---------------------	--

The car starts going down a hill and speeds up to 60 mph. What is the current value of $E(s)$? How does the cruise control change the throttle position?

$E(s)=$ at 60mph	Throttle Position (circle one) <input type="radio"/> Open More <input type="radio"/> Close More <input type="radio"/> Hold Position
---------------------	--

Summary: This exercise has demonstrated how a negative feedback system regulates and stabilizes a system.

The Positive Feedback Case

Now consider the case where the cruise control was incorrectly wired to be the **positive** feedback system illustrated in Figure 3.

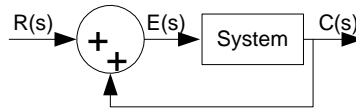


Figure 3: A simple positive feedback system.

For the positive feedback system write the equation for $E(s)$.

$E(s)=$

The incorrectly wired cruise control has been set to 55mph ($R(s) = 55$), and the current speed is 60mph. What is the value of $E(s)$? How does the cruise control change the throttle position (look at the sign of the error signal)?

$E(s)=$ at 60mph	Throttle Position (circle one) <input type="radio"/> Open More <input type="radio"/> Close More <input type="radio"/> Hold Position
---------------------	--

A short time later what has happened to the speed of the car? What has happened to the value and magnitude of the error signal? How does the cruise control change the throttle position? What limits the speed of the car?

$E(s)=$ at 60mph	Throttle Position (circle one) <input type="radio"/> Open More <input type="radio"/> Close More <input type="radio"/> Hold Position
---------------------	--

What would have happened to the speed of the car, with the incorrectly wired cruise control (positive feedback system), if the cruise control had been set to 55mph and the initial speed had been 55.1mph? What if the initial speed had been 50mph?

Adjusting the Time Response with Feedback

A feedback system can also adjust the type of time response a system exhibits. If a system takes a long time to reach its steady state value (over damped), a feedback loop can drive the system to its final value faster. Making it critically or under damped.

Consider the following system:

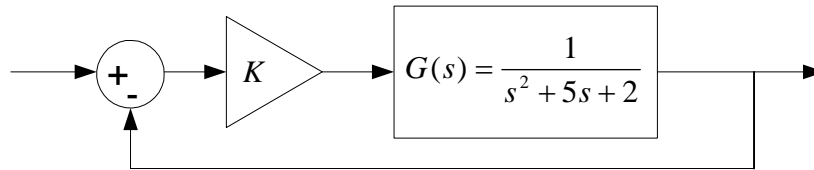


Figure 4: A feedback loop where the error signal is amplified by the gain K .

K is a variable parameter which can be adjusted to change the time response of the system.

1a) Write the characteristic equation for $G(s)$.

Char Eqn for $G(s)$

1b) Find the Closed Loop Transfer Function for the system by simplifying the block diagram.

CLTF=

1c) Write the characteristic equation for the CLTF.

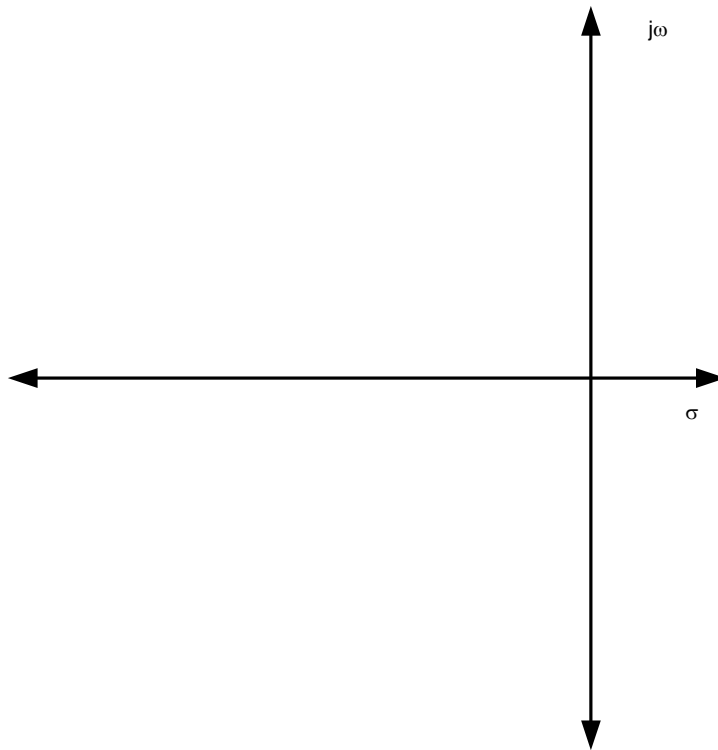
Char Eqn for CLTF

1d) What is the only difference between the characteristic equations found in questions 1a and 1c?

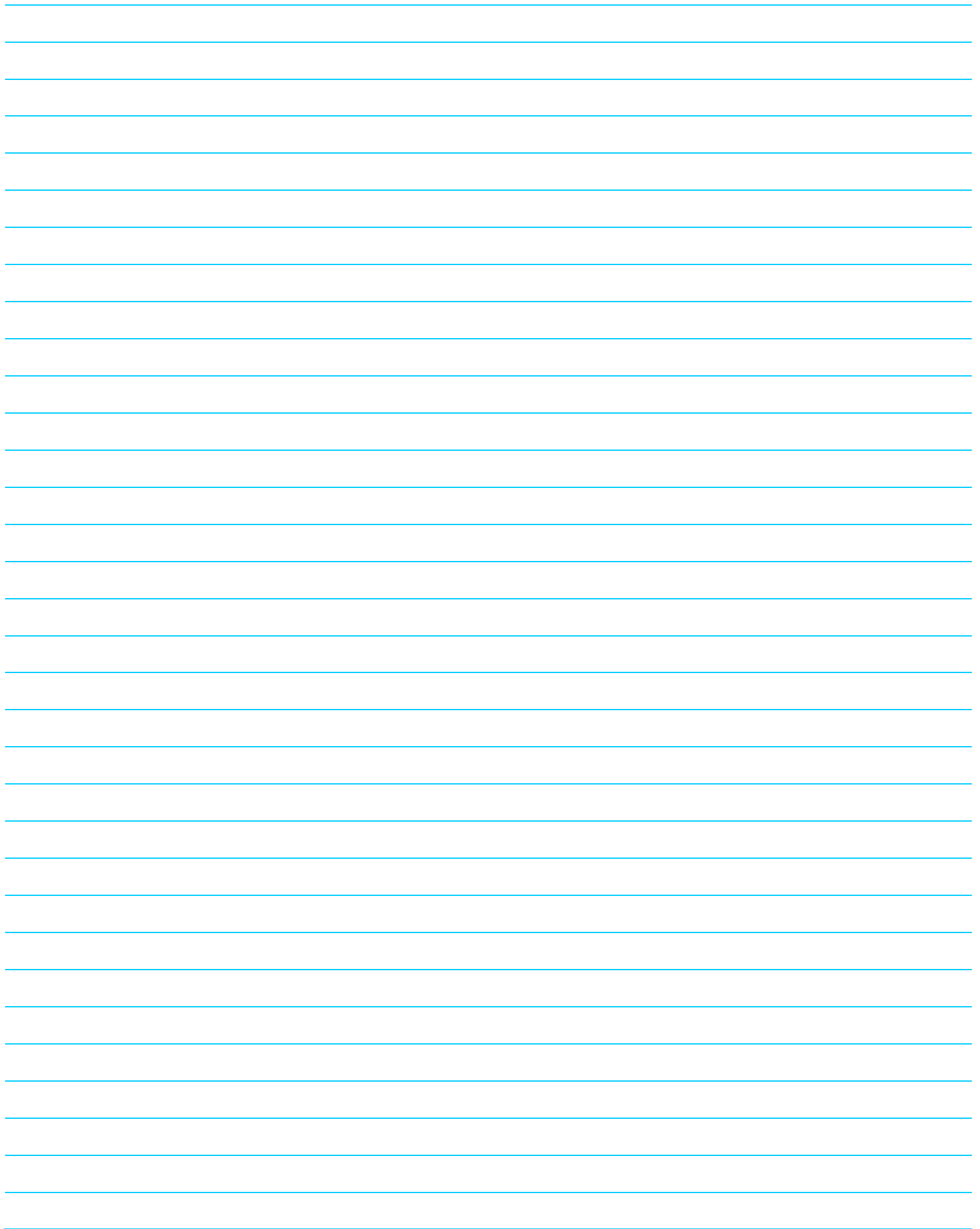
2a) Complete the following table for the specified values of K using the characteristic equation found in question 1c.

K	Roots of Char Eqn	ω_n	ω_d	ζ	Over, Critical, or Under Damped	T_p	T_s	%OS
0								
1								
4								
4.25								
15								
40								

2b) For each of the above values of K, plot the roots of their characteristic equation on the complex plane. Label the roots with their K value.



2c) Will the system ever go unstable? Why or Why not?



Introduction to Controllers

Most feedback mechanisms or circuits will have a controller in them. The controller changes the error signal into a control signal.

A controller uses the error signal to calculate the control signal.
The control signal is applied as the input to the system to change its output.

No Controller

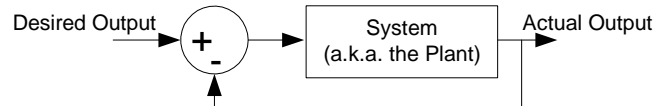


Figure 1: A negative feedback loop with no controller.

Figure 1 shows a negative feedback loop *without* a controller. In this case the error signal is fed directly into the system.

1a) In Figure 1, label the error signal.

1b) How is the error signal calculated?

1c) The controller converts the error signal into the control signal. The control signal is used as the input for the system. Where should the controller be placed in the feedback loop?

Draw a sketch of a feedback loop. Include blocks for the system (plant) and the controller. Label the following: desired output, actual output, error signal, and the control signal.

Proportional Control

The proportional controller is the simplest controller. In proportional control the control signal is the error signal multiplied by a constant value. This value is called the *proportional gain*.

Proportional gain adjust both the transient and steady state responses of the system.

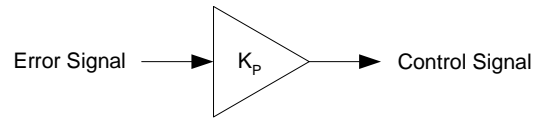


Figure 2: A proportional controller. The control signal is the error signal multiplied by proportional gain, K_p .

2a) Sketch a block diagram of a negative feedback loop with a proportional controller. The system's transfer function is

$$G(s) = \frac{d}{s^2 + b \cdot s + c}.$$

2b) Find the closed loop transfer function for this system.

CLTF(s) =

2c) In the closed loop transfer function, solve for the following in terms of K_P , b , c , and d .

$\omega_n =$	$\zeta\omega_n =$	$\zeta =$
$\omega_d =$		DC gain =

2d) Indicate how the following time response parameters change as the proportional gain, K_P , **increases**.

T_s	T_p	%OS	DC gain
-------	-------	-----	---------

2e) Does the proportional gain, K_P , affect the steady state response of the system? Why or why not?

2f) Does the proportional gain affect the transient response (peak time, percent overshoot, etc.) of the system? Why or why not?

Proportional Derivative (PD) Control

Proportional-derivative (PD) control is achieved by adding a derivative component to a proportional controller.

The derivative component adjusts the transient response of the system.

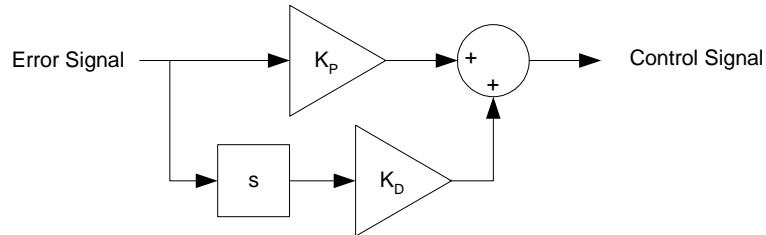


Figure 3: A proportional derivative (PD) controller. The 's' term takes the derivative of the error signal.

The PD controller takes the derivative of the error signal and multiplies it by the **Derivative Gain**, K_D . This derivative term is added to the proportional control signal.

Recall, in the frequency domain, a single 's' term is the derivative operator. If the error signal is changing rapidly then the derivative component will make the control signal large. As the system settles to its final value, the error signal changes more slowly. At this point the derivative component of the controller does not have much effect on the control signal.

3a) Simplify the block diagram of the proportional derivative controller in Figure 3. Label the error signal and the control signal in the answer.

3b) Sketch a block diagram of a negative feedback loop with a proportional derivative controller. The system's transfer function is

$$G(s) = \frac{d}{s^2 + b \cdot s + c}.$$

3c) Find the closed loop transfer function for this system.

CLTF(s) =

3d) Solve for the following in terms of K_D , K_P , b , c , and d .

$\omega_n =$	$\zeta\omega_n =$	$\zeta =$
$\omega_d =$		DC gain =

3e) Indicate how the following time response parameters change as the derivative gain, K_D , **increases**.

T _s	T _p	%OS	DC gain
----------------	----------------	-----	---------

3f) Does the derivative gain, K_D , affect the steady state response of the system? Why or why not?

3g) Does the derivative gain affect the transient response (peak time, percent overshoot, etc.) of the system? Why or why not?

Proportional Integral (PI) Control

Proportional Integral (PI) control is achieved by adding an integral term to a proportional controller.

The integral component adjusts the steady state response of the system and eliminates steady state error.

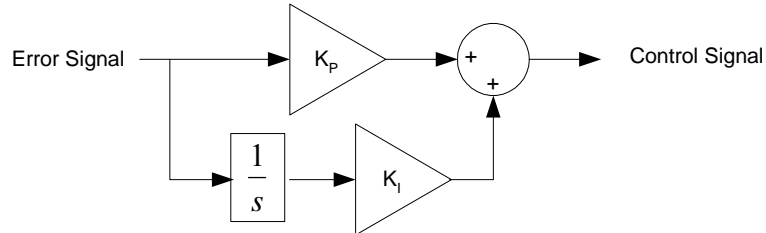


Figure 4: A proportional integral (PI) controller. The '1/s' term takes the integral of the error signal.

While the derivative term only affects the transient responses of the system, the integral term only affects the steady state response.

4a) Simplify the block diagram of the proportional integral controller in Figure 4. Label the error signal and the control signal in the answer.

4b) Sketch a block diagram of a negative feedback loop with a proportional integral controller. The system's transfer function is

$$G(s) = \frac{d}{s^2 + b \cdot s + c}$$

4c) Find the closed loop transfer function for this system. **Do not** have any '1/s' terms in your answer.

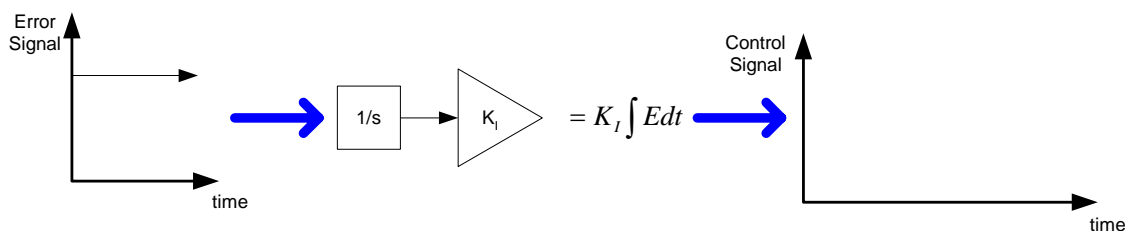
CLTF(s) =

4d) Using the techniques we have learned in this course, we **cannot** solve for the time response of this transfer function. Why not?

The integral term is used to drive the steady state error of the system to zero.

To understand this, assume there is a steady state error of some value E. For example, the cruise control of the car is set to 55 mph and the car is climbing a hill at 50 mph.

The integral term will take the integral of the constant error signal and multiply it by K_I .



4e) Solve the integral $K_I \int E dt$ where E is a constant value for integral component of the control signal and graph it.

Integral Control Signal =

4f) If the steady state error, E, is a large value how does the control signal change? If the steady state error is zero, how does the control signal change?

4g) How does increasing K_I change the control signal? Explain using the graph in 4e.

Proportional Integral Derivative (PID) Control

Proportional Integral Derivative (PID) control is one of the most common forms of control. This controller has all three components discussed previously. This allows the transient responses to be adjusted independently from the steady state response, and the steady state response to be adjusted independently from the transient response.

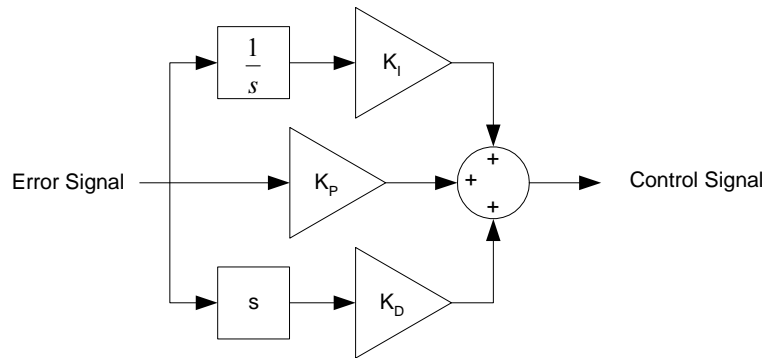


Figure 5: A proportional integral derivate (PID) controller.

5a) Simplify the block diagram of the proportional integral derivative controller in Figure 5. Label the error signal and the control signal in the answer.

5b) Sketch a block diagram of a negative feedback loop with a PID controller. The system's transfer function is

$$G(s) = \frac{d}{s^2 + b \cdot s + c}$$

5c) Find the closed loop transfer function for this system. **Do not** have any '1/s' terms in your answer

CLTF(s) =

Modeling a Gun Turret with Feedback

Build a SIMULINK model to check the results of question 2 in the pre-lab exercise. Build the open loop system model in SIMULINK by doing the following:

Building the Model

- Step 1: Open MATLAB.
- Step 2: Type 'simulink' and hit enter.
- Step 3: Click on the Simulink Library Browser window and then type Ctrl+N to open a new simulation window.
- Step 4: Next find the menu in the bottom left side of the Simulink Library Browser window.
- Step 5: Click on the "Simulink" menu on the left side of the browser, select the "Sources" submenu, select a "Step" block from the right side menu; click and drag it to the blank Simulink project window.
- Step 6: Click the "Math Operations" submenu in the Library window, and find the 'sum' block; drag it to the project window. Double click on it and change the list of signs to "+ -". Click OK.
- Step 7: Go to the "Math Operations" submenu and find the Gain block. Click and drag this block to the simulation model window.
- Step 8: Next look under "Continuous" submenu in the Library Browser window and find the 'Transfer Fcn' block; drag it to the project window.
- Step 9: Now look under the "Sinks" submenu in the Library Browser window and find the 'Scope' block; drag it to the project window.
- Step 10: Align the blocks from left to right in the order shown in Figure 1.
- Step 11: Click on the small triangle on the right of the Step block then drag out a line to the "+" input on left side of the summing block. Repeat this for the Summing block, the gain block, the transfer function, and the scope. Now, all the blocks should be connected in a line....it should look like this:

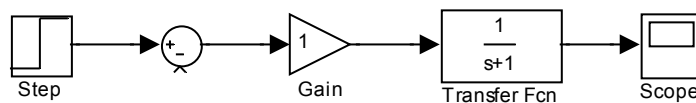



Figure 1: The SIMULINK model of the open loop system.

- Step 12: Double click on the 'Transfer Fcn' block. **Enter the coefficients of the transfer function from question 2 of the pre-lab exercise in the numerator and denominator lines** in the form of [b c d] where bs^2+cs+d . Ensure spaces are placed between each coefficient value. Click OK.
- Step 13: Double click on the Step block. Enter **the value of 0 for 'Step Time'**. Click OK. Double click on the gain block and enter a **value of 6 for the gain** (since $f(t)=6u(t)$). Click OK.
- Step 14: Hit Ctrl+E. This will bring up a Simulation Parameters box. **In the 'Stop Time' block, set this value to 20.0. Adjust the Relative Tolerance to 1e-6.** Click OK.
- Step 15: Finally, click on the play button  or press Ctrl+T to run the simulation.
- Step 16: Double click on the scope block to view the step response plot.

Labeling, Saving, and Printing Steps:

- Step 1: Double click on the scope in the simulation window.
- Step 2: Click the ‘Autoscale’ icon (i.e. binoculars) in the header of the “Scope” screen.
- Step 3: Right click on the scope plot in the “Scope” screen and bring up the ‘Axes properties’ menu. **Enter 0 for Ymin and 2.5 for Ymax.** For the title, enter ‘(YOUR NAMES)—MASS SPRING DAMPER—(THE DATE)’.
- Step 4: Save (to your own memory device, i.e. disk) and **PRINT both the plot and the simulation model (i.e. block diagram).**
If printed plot is unreadable (i.e. too light), go to the “general” settings in the preferences window (“File” menu of the **SIMULINK** window → “Preferences...”). Set “Figure Window Printing” to “**Always send as black and white.**”
- Step 5: NEATLY, mark/label (using straight lines) by hand, using a pencil:
 t_p = your value, t_s = your value, %OS = your value,
 and the steady-state output X_{ss} = your value...on the plot. Also, label the X-axis as “TIME (sec)” and the Y-axis as “x(t) DISPLACEMENT in meters.”

Does the graphed response match the pre lab calculations?

A More Realistic Gun Turret Model

Now consider some realistic weapon systems. Many weapon systems such as gun turrets or rotating antennas are rotating mass-damper systems: they rotate and naturally have mass and friction. Although these are rotating systems, vice translating systems like the above mass-spring-damper, they are modeled similarly (but use different variables). But they do not have springs countering the rotation.

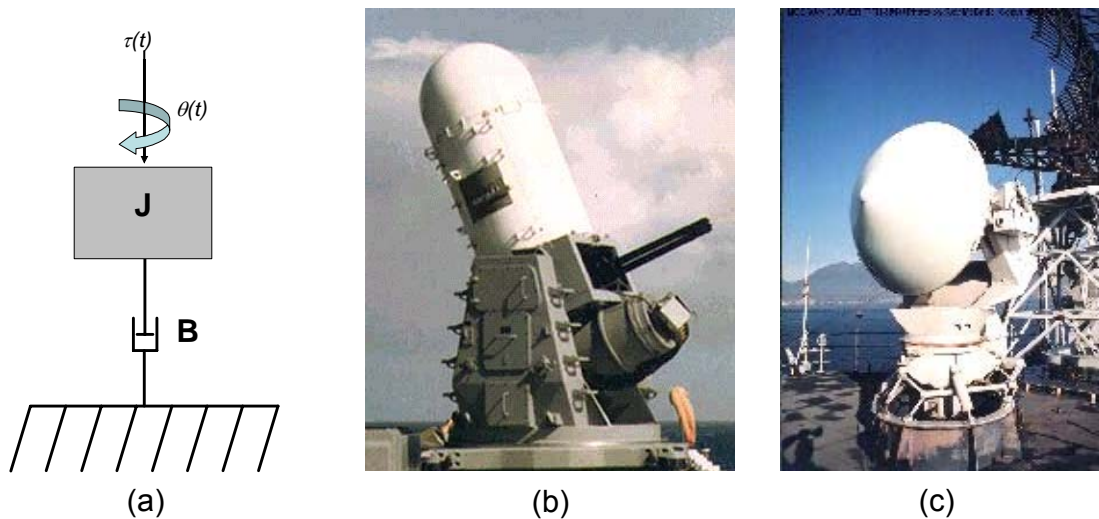


Figure 2: A simple representation of the gun turret (a), an actual gun turret (b), and an antenna mount (c).

1) Write the transfer function for a rotational mass spring damper. Hint: See Lesson 3b.

G(s)=

Lesson 7: Modeling a Gun Turret w/ Feedback

For this simulation use the following parameters: $\mathbf{J}=2000\text{N}^{-1}\cdot\text{m}^{-1}\cdot\text{s}^{-2}$ $\mathbf{B}= 1000 \text{ N}\cdot\text{s}$
 $\boldsymbol{\tau}(\mathbf{t})= 2000\mathbf{u}(\mathbf{t}) \text{ N}\cdot\text{m}$ (a step input). The gun turret has no spring.

2) Write the turret's transfer function, substituting in the simulation parameters.

$G(s)=$

3) If you attempt to calculate θ_{ss} , the steady state position of the gun turret, in response to a step input of torque, you get an unusual solution. Calculate it here:

$\theta_{ss} =$

4) Use your intuition to explain why this happens. If you apply a constant torque to a rotational mass with drag, will it stop at some particular position? Will it stop at all? Why is this different than a mass-spring-damper system?

Using SIMULINK to Investigate the Model

5) Simulate the step response of this open-loop system using the model developed in question 2.

Step 1: Double click on the 'Transfer Fcn' block. Enter the coefficients of the transfer function from question 2 into the numerator and denominator in the form of [b c d] where bs^2+cs+d . Ensure spaces are placed between each coefficient value. Click OK.

Step 2: Double click on the gain block and enter 2000 [since $\tau(t)=2000u(t)$]. Click OK. Click and drag on the gain block to make it larger. This will allow you to see the gain value.

Step 3: Finally, click on the play button  or press Ctrl+T to run the simulation.

Plot the result as follows:

Step 1: Double click on the scope in the simulation window.

Step 2: Click the 'Autoscale' icon (i.e. binoculars) in the header of the "Scope" screen.

Step 3: Right click on the scope plot in the "Scope" screen and bring up the 'Axes properties' menu. For the title, enter '(YOUR NAMES)—Gun Turret Step Response Without Feedback—(THE DATE)'.

Step 4: **Print both the plot and the simulation model (i.e. block diagram).**

Lesson 7: Modeling a Gun Turret w/ Feedback

As seen above, the **open-loop** gun turret system is **very difficult to control** and aim steadily at a particular point, making it ineffective as a weapon system. **This can be corrected by adding feedback.** The closed loop transfer function obtained by using feedback will behave like a mass spring damper. The proper selection of the set point, proportional, and derivative gains (three adjustable parameters: K_{sp} , K_p , and K_D) will allow you to specify and meet the performance criteria for the system. The criteria are:

- **θ_{ss}** : To be an effective weapon system, the gun must train to the desired position and settle there.
- **Settling Time**: To be effective, it must get there expeditiously, consider the significance for CIWS.
- **%OS**: A wildly oscillating gun turret is going to be less accurate.

Now, we will modify the gun turret model to achieve the following system response specifications:

$$\theta_{ss} = 1.0 \text{ radian} \quad t_s < 1 \text{ sec} \quad \%OS < 10\%$$

6a) What is the CLTF for the system illustrated in Figure 3a? Hint: The answer to question 3 in the pre-lab exercise is a good starting point.

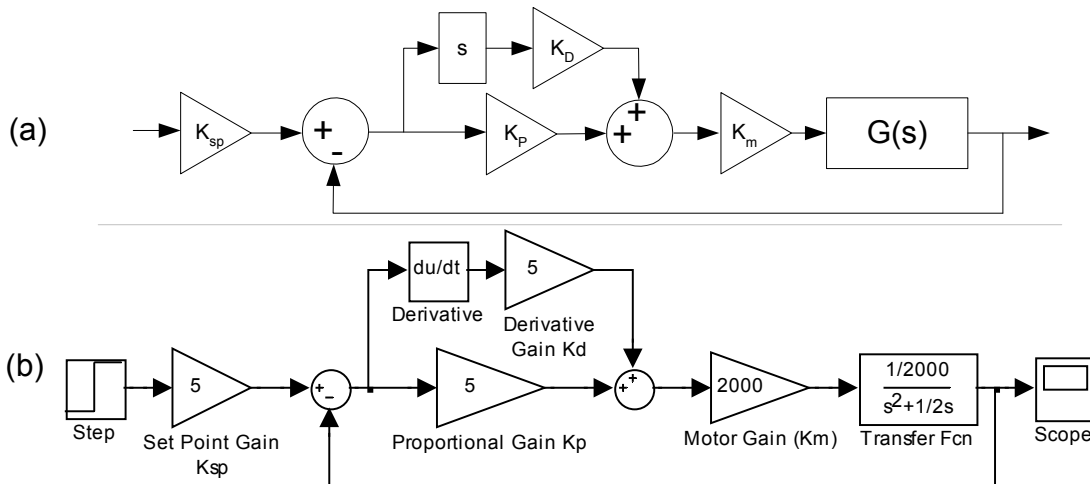


Figure 3: The system block diagram (a) and the SIMULINK model (b).

CLTF=

6b) Use the closed loop transfer function to explain which of the three gains (K_{sp} , K_p , or K_D) is the only gain that will affect the system's steady state value. This is the gain you should change to set the gun's final position. Why don't the other two gains affect the gun's final position?

Lesson 7: Modeling a Gun Turret w/ Feedback

7) Now we are going to use SIMULINK to iteratively find the controller gain values (K_{sp} , K_P , and K_D) which will meet the above requirements. Pick a value for each feedback constant, and adjust them until the above performance requirements are met. Write the final values in the spaces below. **Do NOT get your values from other lab groups.**

Using SIMULINK, model the system. Your block diagram should include the original blocks the pre-lab model, **and will look like the diagram in Figure 3b** after the following steps 1 to 12 below. Your values for the forward path gain, feedback gains and the transfer function should have been inserted. To see the values in the boxes, expand the boxes horizontally by clicking and dragging a box corner.

- Step 1: Label the gain block to the left of the transfer function block 'Motor Gain (K_m)'.
- Step 2: Click, then delete, the line from the sum block to the motor gain block .
- Step 3: Click on the Library Browser window.
- Step 4: Go to the "Math Operations" submenu and find the **Gain** block. Click and drag this block to the simulation model window three (3) times. Alternatively, you can use the copy and paste commands to duplicate the gain block.
- Step 5: Go to the "Math Operations" submenu and find the **Sum** block. Click and drag this block to the simulation model window. Double click on the sum block and set the list of signs to '+ + |'. Move the this sum block to the left of the motor gain block.
- Step 6: Click on the "Continuous" submenu in the Library Browser window and click on the Derivative. Drag one of these blocks to the simulation window.
- Step 7: Place two gain blocks between the sum blocks, one below the other.
- Step 8: Place the derivative block to the left of the upper Gain block.
- Step 9: Place the last Gain block between the Step block and first Summing junction.
- Step 10: Connect the blocks as illustrated in Figure 3b.
- Step 11: Double click on the Gain blocks and enter your estimated values.
Left click on the three gain title blocks and properly rename them as "Set Point Gain K_{sp} ", "Proportional Gain K_p ", and "Derivative Gain K_d " as appropriate.
- Step 12: Adjust your 'Simulation Parameters' using Ctrl+E:
Stop Time = 3 seconds, Relative Tolerance = 1e-6.

Lesson 7: Modeling a Gun Turret w/ Feedback

Run the simulation and verify your system meets the specs. It should, very closely. If not, recheck your math, correct errors, and try again. **Once it meets the specs, print the results:**

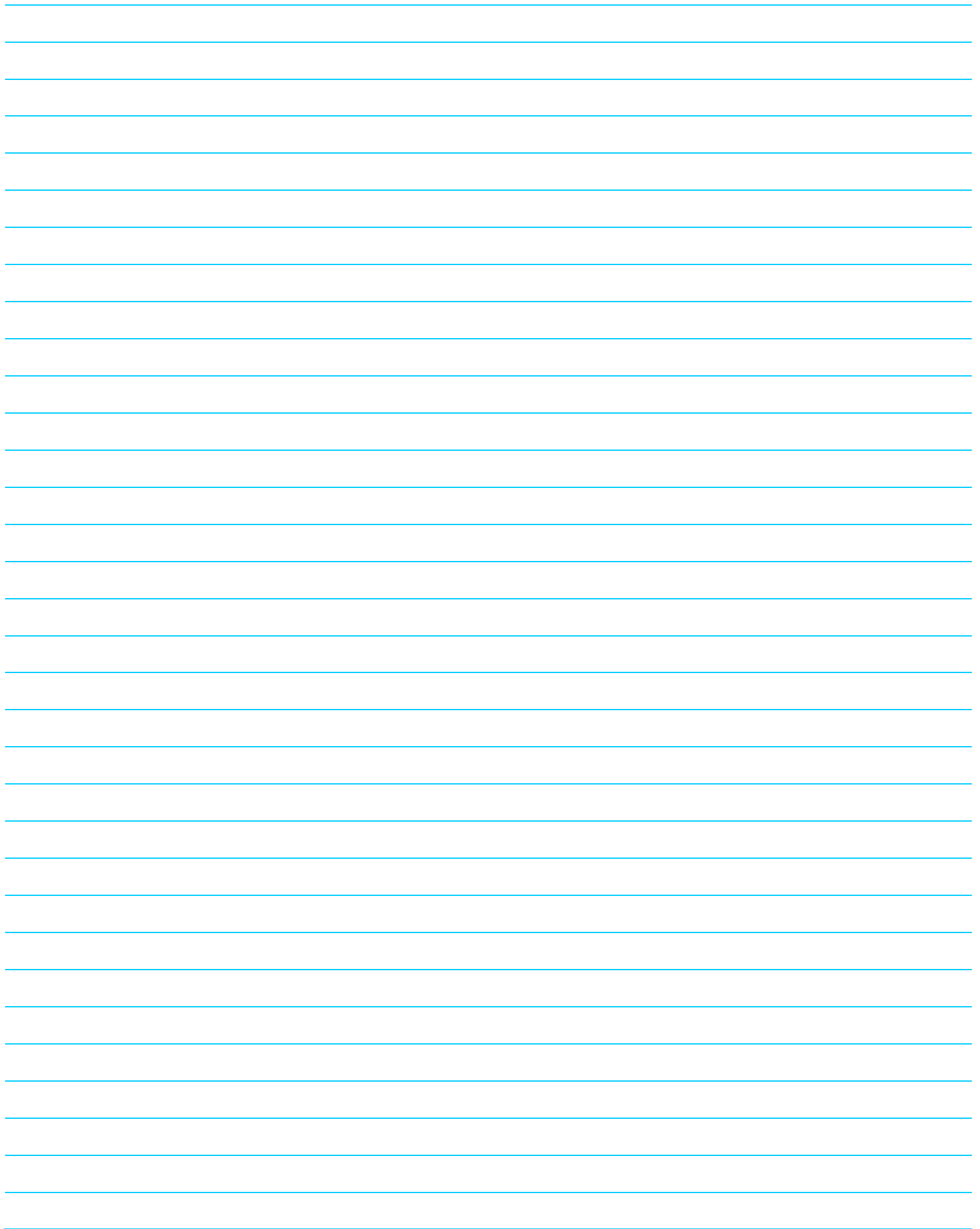
- Step 1: Double click on the scope in the simulation window.
- Step 2: Click the 'Autoscale' icon (i.e. binoculars)) in the header of the "Scope" screen.
- Step 3: Right click on the scope plot in the "Scope" screen and bring up the 'Axes properties' menu. For the title, enter '(YOUR NAME)—System Response With Feedback—(THE DATE)'. Click OK>
- Step 4: **PRINT both the plot and the simulation model (i.e. block diagram). Your name should be typed on the block diagram print out.**
- Step 5: NEATLY, mark/label (using straight lines) by hand using a pencil:

t_p = your value, t_s = your value, %PO = your value, and the steady-state output θ_{ss} = your value...on the plot. Also, label the X-axis as "TIME (sec)" and the Y-axis as " $\theta(t)$ in radians."

$K_{sp} =$
Set Point Gain

$K_p =$
Proportional Gain

$K_D =$
Derivative Gain



Correcting for Disturbances

Control systems are designed to achieve a specific output even when the system is in the presence of disturbances. Stabilizing a gun barrel on a moving platform is a classic example of this situation. As the vehicle pitches and rolls, the control system should be able to hold the barrel at a constant elevation.



Figure 1: The Mark 45 5 inch gun (a) and M1A2 Main Battle Tank (b) both have stabilized gun barrels. The relationship between barrel elevation, platform roll, and total elevation angles (c).

Conceptually, all gun barrel elevation systems are very similar. The desired barrel elevation is an input to the controller¹. The controller generates a control signal to drive the actuator. Usually, the actuator is either a hydraulic or electric motor. The actuator applies a torque to change the barrel's elevation. Figure 2 depicts this process.

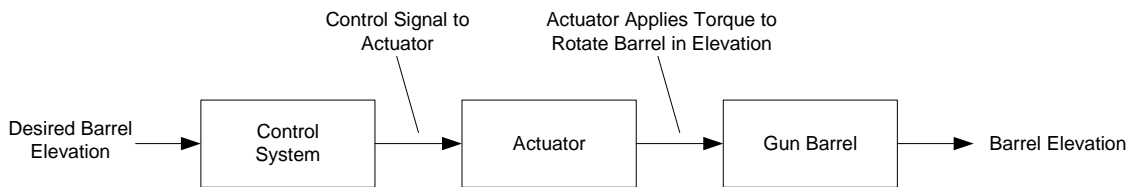


Figure 2: The major components of the barrel elevation system.

The model in Figure 2 does not account for the platform rolling or pitching. Any platform roll or pitch will be added to or subtracted from the barrel's elevation, see Figure 1c.

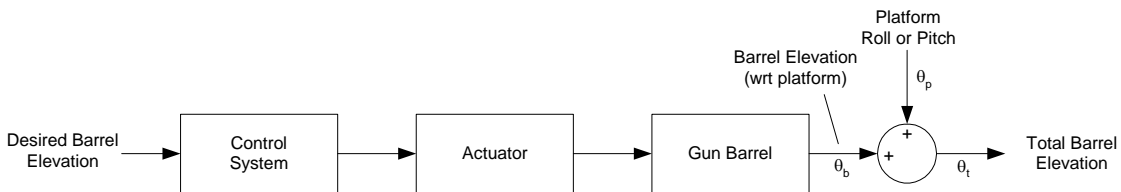


Figure 3: A model of the barrel elevation system with platform roll/pitch affecting the output.

1a) Do the models in Figures 2 and 3 depict an open loop or closed loop control system?

Open Loop
Closed Loop

¹ The desired input could be entered manually or come from a targeting computer.

The system needs feedback to counter the effects of the platform's roll and pitch.

1b) Sketch the system illustrated in Figure 3 with a feedback path and elevation error as an input to the controller. Which signal did you select to feedback? Why?

The Gun Barrel's Transfer Function

Each of the blocks in Figures 2 and 3 represent a model of a system component.

2a) The gun barrel can be represented by a second order mechanical rotational system. Write a transfer function which relates barrel *angle* to *applied torque*.

Hint: See Lesson 3b.

G(s)=

2b) Use the following values for this simulation: $J = 200$, $B=100$. There is no spring in the barrel. Write the gun barrel's transfer function, substituting in the simulation parameters.

G(s)=

The Actuator Model

The simplest model of the actuator is one where the amount of torque generated is proportional to the control signal.

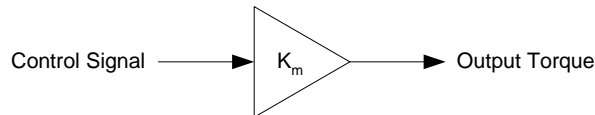


Figure 4: A very simple actuator model in which the torque generated by the actuator is proportional to the control signal. K_m is the motor gain.

3a) If the motor gain (K_m) has a value of 200, complete the table below.

Control Signal Value	Actuator Output Torque	Control Signal Value	Actuator Output Torque
50		0	
-5		100	

3b) Does this actuator model impose any limits on the torque generated by the motor?

The Complete SIMULINK Model

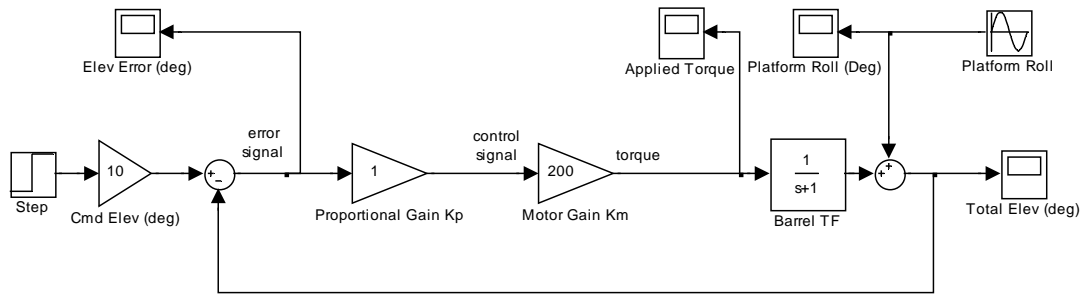


Figure 5: The SIMULINK model of the gun barrel elevation control system with a proportional controller.

Figure 5 illustrates the SIMULINK model for the gun barrel elevation control system. This model is available in the file **gun_barrel_elev.mdl**. Ensure this file is in your MATLAB working directory. A detailed discussion of the model follows.

The “Step” and “Commanded Elevation” blocks provide an input to the control system. Initially the gun barrel is at an elevation of zero degrees. At the start of the simulation (time = 0) the commanded elevation for the gun is changed to the value in the “Cmd Elev” block.

3c) In Figure 6 label the controller and the actuator. What kind of controller is shown in Figure 6?

The “Barrel TF” block is the transfer function for the gun barrel. This transfer function has torque as an input and barrel elevation angle as its output.

Any platform roll is added to the barrel elevation to calculate the total barrel elevation. The total barrel elevation is used as the feedback signal.

Preparing the Model for Use

1. Enter the transfer function you found for the gun barrel (question 2b) in the model’s barrel transfer function block.
2. Verify the parameters for the step block are: Step Time = 0, Initial Value = 0, Final Value = 1.
3. Verify the “Cmd Elev” block has a value of 10.
4. Verify the following gain values are used: $K_p = 100$, $K_m = 200$.
5. Verify the simulation stop time is set to **20** seconds in the “**Configuration Parameters...**” window (in the “**Simulation**” menu).
6. Verify the Amplitude of the platform roll is set to **0**. Double click on the platform roll block.

Testing the Model with No Platform Roll

4a) Make a sketch of the complete model. You do not need to include the “scope”, “step”, or “Cmd Elev” blocks in your sketch.

4b) **Assuming there is NO platform roll**, find the closed loop transfer function for the system sketched in question 4a. No platform roll means the platform roll block and its summing junction can be ignored when finding the CLTF. Additionally, do not include the step input block or “commanded elevation” block.

CLTF(s)=

4c) Solve for the following, using the closed loop transfer function found in question 4b.



Response Type

T_s

%OS

4d) Calculate the steady state gun barrel elevation for a command elevation of 10 degrees applied as a step input. Hint: Use the final value theorem.

θ_{ss}

4e) Verify your answers to questions 4c and 4d with the SIMULINK model. Run the model by clicking the  button in the model window. Display the plots of Elevation Error, Applied Torque, and Barrel Elevation by double clicking on the scope blocks in the model. You may need to click the  button in the plot windows to set the axis limits properly. Estimate the following values from the plots.

Response Type	T_s	%OS
---------------	-------	-----

4f) Did the simulation match your calculations? Does this seem like good performance for a gun barrel with NO platform motion?

A Better Controller (proportional derivative)

To improve the response of the gun barrel replace the proportional controller with a proportional derivative controller. Hint: See Lesson 6b.

5a) Make a sketch of the complete model with PD control. You do not need to include the “scope”, “step”, or “Cmd Elev” blocks in your sketch.

5b) **Assuming there is NO platform roll**, find the closed loop transfer function for the system sketched in question 5a. No platform roll means the platform roll block and its summing junction can be ignored when finding the CLTF. Additionally, do not include the step input block or “commanded elevation” block. **Leave K_P and K_D as variables.**

CLTF(s)=

5c) Using the closed loop transfer function found in question 5b and $K_P = 100$ and $K_D = 10$, solve for the following:

Response Type	T_s	%OS
---------------	-------	-----

5d) Calculate the steady state gun barrel elevation for a command elevation of 10 degrees applied as a step input. Hint: Use the final value theorem.

θ_{ss}

5e) Run the simulation again with the PD controller. There should be a large improvement in the system's response. **Print the Total Elevation plot for your notes.** Estimate the following from the plot.

Response Type	T_s	%OS
---------------	-------	-----

5f) Do these results match the results calculated in questions 5c and 5d? Has the performance of the gun barrel improved?

5g) Sketch the plot of Applied Torque for this response below. What is the maximum torque applied to the gun barrel? Why did the simulation predict such a large initial torque value?

Maximum Applied Torque

A Better Actuator Model

Real actuators have an upper limit on how much torque they can apply. It does not matter how big the control signal is, the motor just can't push any harder. Add this feature to the model with a saturation block. The saturation block is found in the SIMULINK Library Browser in the "Commonly Used Blocks" or "Discontinuities" menu. Place the saturation block after the motor gain block as illustrated in Figure 6. Double click on the saturation block and set the upper and lower limit to 3000 and -3000 respectively.

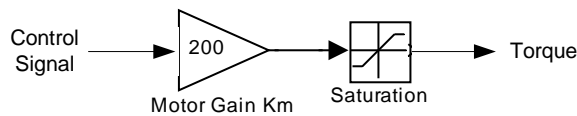


Figure 6: An improved actuator model which limits the torque to a maximum value.

6a) Run the simulation again. View the Applied Torque plot. Change the Y axis limits to ± 4000 by right clicking on the Y axis. Use the button to zoom in on the first 6 seconds of the simulation. **Print and label this plot.** Describe what is happening to the applied torque below.

6b) How did settling time change? Why?


Testing the Model with Platform Roll

Initially apply a roll of ± 10 degrees and a period of 8 seconds to the platform. Double click on the platform roll block and set the amplitude to 10 and set the frequency to $\pi/4$. $\pi/4$ may be entered as “pi/4”.

Table 1 lists several periods and their corresponding frequencies in radians per second.

Period [sec]	Frequency [rad/sec]
2	π
4	$\pi/2$
6	$\pi/3$
8	$\pi/4$
10	$\pi/5$

Table 1: Periods and Frequencies in radians per second

7a) Run the simulation again. How much elevation error does the ± 10 degree roll introduce? What is the maximum torque applied by the actuator? How much torque is applied to cancel out the platform roll? Use the  button to magnify the Y axis to accurately read the error and torque values.

Roll Induced Elevation Error	Maximum Applied Torque	Torque Required to Correct for Platform Roll
---------------------------------	---------------------------	---

7b) Run the simulation again with a **4** second period, ± 10 degree roll, and simulation stop time of 60 seconds. Sketch a plot of the applied torque below. What is the roll induced error? What has gone wrong?

Roll Induced Elevation Error

Lesson 8 Disturbances and Actuator Limitations

7c) Set $K_P = 100$ and $K_D = 10$. Set the maximum torque of the motor to 9,000 Nm, by changing the upper and lower limits for the saturation block. Re-run the simulation. How does this improve the response, and why?

7d) Based on the results of this lab, what are two methods of improving a system's ability to compensate for disturbances? Why?

Lesson 9: PID Controllers

The Generator Set Model

The goal of this lesson is to control the frequency output of an AC generator set as the electrical load placed on the generator changes.¹

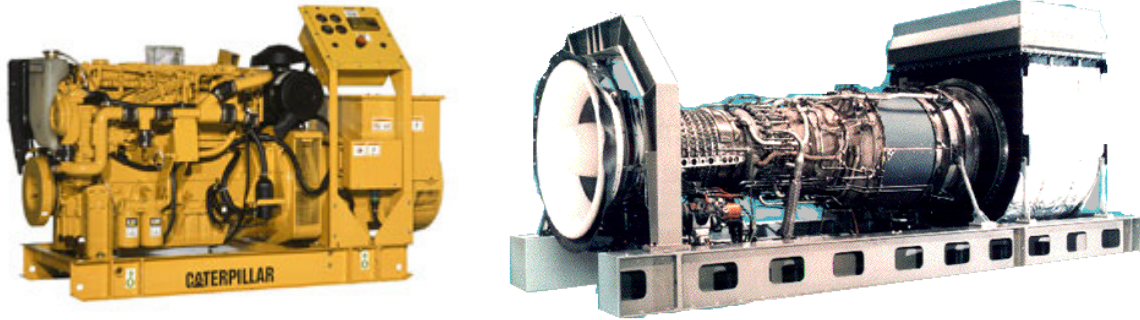


Figure 1: A Caterpillar diesel generator set and a General Electric LM2500 gas turbine generator set.

1a) What determines the output frequency of an AC generator?

1b) What determines the required torque of an AC generator?

1c) As the loading of the generator increases, how does the required torque to rotate the generator change? What happens to the generator's speed?

In this simplified model, the generator is rotated by an engine which is controlled by a controller.

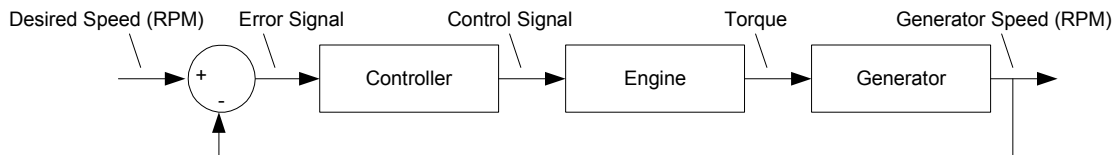


Figure 2: The conceptual diagram of the model to control generator speed.

The engine is modeled as a 1st order system with the transfer function

$$G_E(s) = \frac{100}{s + 1}.$$

¹ The term “generator set” refers to the whole machine (i.e. controller, engine, and generator). The term “generator” refers to just the device which converts rotational motion to electricity.

Lesson 9: PID Controllers

The electrical generator is modeled as a 1st order rotational mechanical system with the transfer function

$$G_G(s) = \frac{\frac{1}{J}}{s + \frac{B}{J}}$$

where J is the rotational inertia of the generator and B is the friction coefficient for the generator. The generator's transfer function has torque as its input and rotational speed as its output. The output frequency of the generator is determined by its rotational speed. The control system should maintain the required generator speed, and thereby output frequency, under a wide range of loading conditions.

As a larger electrical load is placed on the generator, more torque is required to keep the generator spinning at the same speed. This is analogous to adding more friction into the system. In a mechanical rotational system the relationship for friction is

$$\tau = B \omega$$

where ω is rotational speed, B the frictional coefficient, and τ is torque. Adding more friction to the system increases the magnitude of B. In this model there are two sources of friction in the generator. The internal friction of the generator, B_G , and the "friction" created by the electrical load on the generator, B_L . Specifying both of the frictional terms in the generator's transfer function yields

$$G_G(s) = \frac{\frac{1}{J}}{s + \frac{B_G + B_L}{J}}$$

The block diagram for this transfer function is drawn with the frictional terms as separate blocks. This makes it easier for the simulation to vary the friction due to electrical loading, B_L .

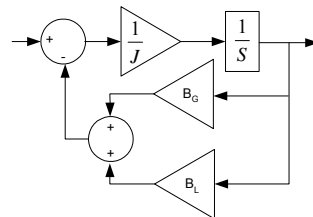


Figure 3: The block diagram of the generator transfer function.

2) Simplify the block diagram in Figure 3 to show that it is equivalent to $G_G(s)$ listed above. Show the intermediate diagrams.

G(s)=

The Open Loop Model of the Generator Set

Imagine there is no feedback and no controller in the generator set. The throttle on the engine is set to 1800 RPM and never changed.

3a) What will happen to generator speed as loading is increased? Why?

3b) Sketch an open loop block diagram with the appropriate transfer functions for the engine and generator. $J = 1000$ and $B_G = 100$. Leave B_L as a variable.

3c) Simplify the block diagram and write the resulting transfer function.

3d) Using the final value theorem, calculate the final values for the open loop system. The input is a step input of 1800 RPM. What happens to generator speed as loading is increased? Why?

B_L	Final Value
0	
10	
100	
1,000	
100,000	

3e) Do the results in question 3d match your prediction in question 3a? How effective is open loop control in this application?

Proportional Control

4a) Sketch a proportional controller. Label the error signal and control signal. Hint: See Lesson 6b.

4b) How is the error signal for the controller generated?

4c) How is the control signal generated? Hint: See Lesson 6b.

4d) Sketch a block diagram of the complete generator set model with a proportional controller and the transfer functions for the engine and generator. Use the generator transfer function model from Figure 3. **$J = 1000$ and $B_G = 100$** . Leave B_L as a variable.

4e) Simplify the block diagram in question 4d to find the closed loop transfer function for the system. Hint: Use the solution found in question 2 of the pre-lab with K_D set to zero.

Lesson 9: PID Controllers

4f) From your answer in 4e, calculate the proportional gain (K_p) required to keep the generator speed at full load ($B_L=100,000$) within 5% of its initial value. This means the DC gain of the closed loop transfer function found in 4c should be at least 0.95.

$K_p =$

The SIMULINK Model of the Generator Set

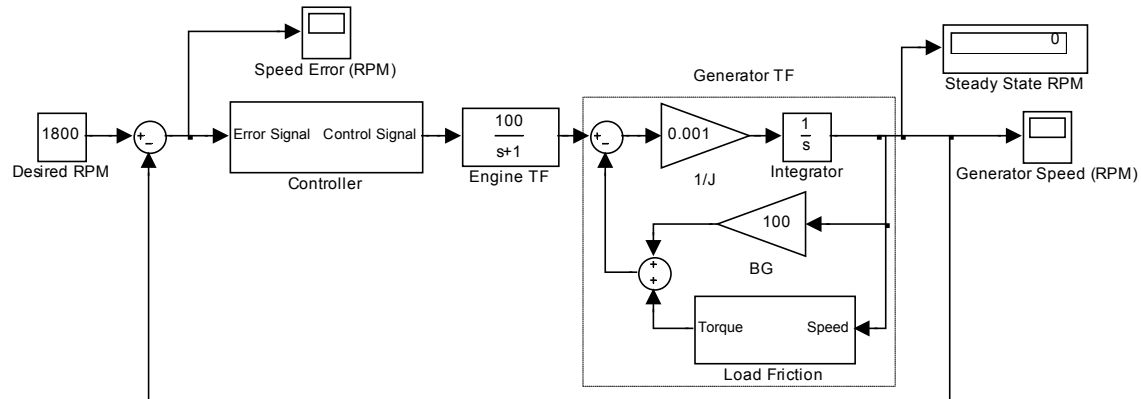


Figure 4: The SIMULINK mode of the generator set.

Figure 4 depicts the SIMULINK model of the generator set. The generator’s transfer function is modeled as described in Figure 3. This allows the internal friction, B_G , and the load friction, B_L , to be modeled separately. The controller and load friction are both implemented as subsystems in SIMULINK.

The Load Friction Subsystem

The load friction subsystem models the equation

$$\tau = B_L \omega .$$

The subsystem hides modeling required to allow the simulation to vary the load with time. This simplifies the overall view of the model. Double clicking the load friction subsystem displays its contents.

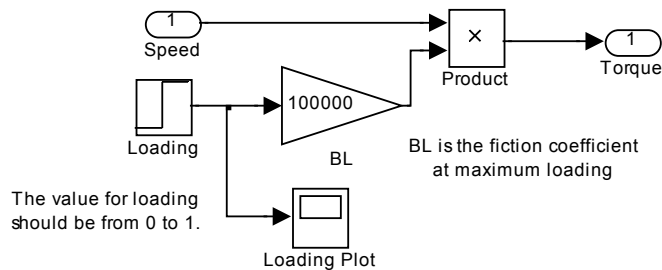


Figure 5: The load friction subsystem. This subsystem allows the simulation to vary the load friction with time.

The value for B_L is the frictional coefficient at maximum loading. For this simulation B_L has a value of 100,000. This is a thousand times larger than the generator’s internal friction of 100. At full load the generator will be 1000 times more difficult to turn. When the “Loading” step function has a value of 0, no load is applied to the system. Maximum load is applied to the system when the step function has a value of 1.

Preparing the Model for Use

1. Download the model file **generator.mdl** to your work area and open it in SIMULINK.
2. Set Simulation Stop Time to **20** seconds (Set in the Simulation Menu → Configuration Parameters...).
3. Set the following values: Desired **RPM = 1800**, **BG=100**, **1/J= 0.001**
4. In the “Load Friction” subsystem:
 - a. $BL=100,000$
 - b. Loading Step Input: **Initial Value = 0**, **Final Value = 1**, **Step Time = 10**

Testing the Proportional Controller

4g) Verify the answer to question 4d with the SIMULINK model of the generator. After setting the model up for use, implement a proportional controller in the “Controller” subsystem. Use the gain value calculated in question 4f. **Print the “Generator Speed” plot for your notes.**

- Label it as “Generator No Load Step Response”.
- Annotate your K_p value on the plot.
- Annotate the point on the plot where full load is applied.
- What is the steady state generator speed at the end of the simulation? Is this within 5% of the desired RPM of 1800?

Final RPM=	$K_p =$
------------	---------

4h) Initially the generator set is shutdown (0 RPM). At time zero the generator set gets started and the controller immediately tries to regulate generator speed to 1800 RPM. How good is the transient response between 0 and 10 seconds? What is the response type?

4i) After full loading is applied, at 10 seconds, the generator’s output is much smoother than it was for the first 10 seconds of the simulation. Why?

4j) Try to improve the transient response by lowering the gain K_p in the controller. Does this improve the transient response? As K_p is lowered, what happens to the steady state response (the final generator speed) at full load?

Proportional-Derivative (PD) Control

5a) Sketch a proportional derivative controller. Label the error signal and control signal.

5b) Write an expression for the control signal in terms of the error signal?

5c) Sketch a block diagram of the complete generator set model with a **proportional-derivative** controller and the transfer functions for the engine and generator. Use the generator transfer function model from Figure 3. **$J = 1000$ and $B_G = 100$** . Leave B_L as a variable.

5d) Simplify the block diagram in question 5c to find the closed loop transfer function for the system. Hint: Use the answer to question 2 of the pre-lab exercise.

Lesson 9: PID Controllers

5e) Implement a proportional derivative controller in the SIMULINK model. Adjust the gain terms K_P and K_D to improve the generator set's response (both at startup and under load). Most real systems have gain values less than 1000. Ensure both the proportional and derivative gains are less than 1000. Record the gain values and steady state speed. **Print the "Generator Speed" plot for your notes.** The "Generator Speed" plot should be scaled to show the whole response.

Final RPM=	K_P =	K_D =
------------	---------	---------

5f) Use the **final value theorem to calculate the steady state generator speed under full load.** Verify this result with the simulation.

Final RPM=

5g) As the proportional gain is lowered, how does the steady state generator speed change? As the derivative gain is lowered, how does the steady state generator speed change? How does the transient response change as each gain is lowered?

	Affect on Steady State Response.	Affect on Transient Response.
$K_P \downarrow$		
$K_D \downarrow$		

Proportional-Integral-Derivative (PID) Control

The previous section demonstrated that adding a derivative component to a controller allows the transient response to be adjusted independently from the steady state response. However, a PD controller **cannot** adjust the steady state response without also affecting the system's transient response.

Adding an integral term to the controller will allow the controller to adjust the steady state response of the system independently of the transient response.

How Integration Removes Steady State Error

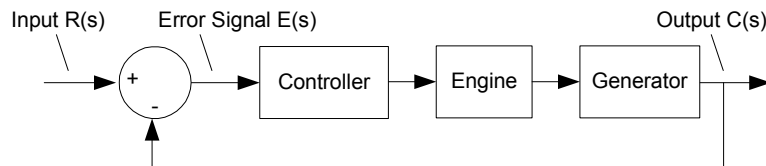


Figure 6: A simple negative feedback system with the input, output and error signals labeled.

- 6a) Write the equation for the error signal, $E(s)$, in terms of the input, $R(s)$, and the output, $C(s)$, for the simple negative feedback system depicted above.
- 6b) If the output signal, $C(s)$, equals the input signal, $R(s)$, what is the value for the error signal, $E(s)$? In the generator example, this corresponds to the steady state speed equaling the desired speed.

Lesson 9: PID Controllers

If there is a constant error in the system's output, i.e. the generator's steady state speed does not equal the desired speed, a plot of the error signal would be similar to Figure 7.

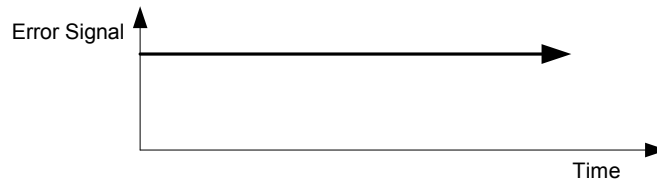
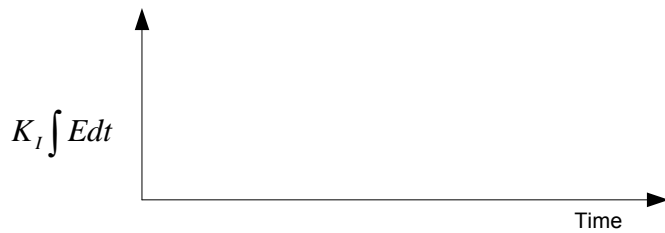
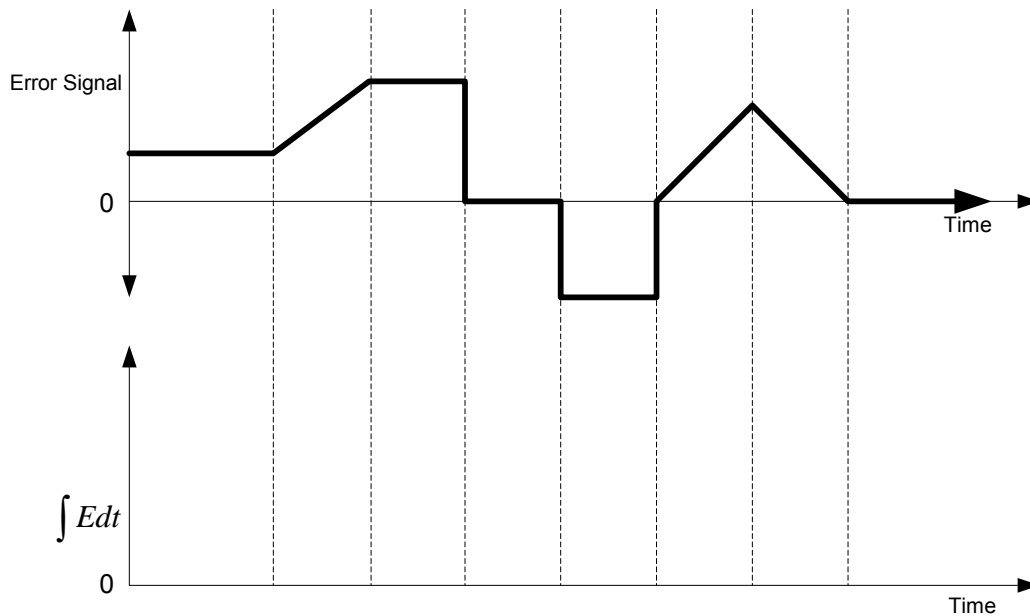


Figure 7: A plot of a constant error signal.

6c) Solve $K_I \int E dt$, where E is a constant value of the error signal and K_I is the integral gain. Plot this integral on the axes below? How does the value of K_I and the magnitude of E change the shape of the graph?



6d) Sketch the integral of the following error signal on the axes below.



The previous questions demonstrate that when the error signal is not zero the integral term will be changing. **The integral term of the PID controller will keep adjusting the control signal until the error signal becomes zero.**

An Alternate Integral Term Explanation

At steady state all signals in the system are constant and unchanging, including the output of the controller's integral term. Therefore, the input to the controller must be **zero**, because only a zero input will cause an integral to not change. Since the input to the controller is the error signal, the error signal must be zero at steady state.

PID Control of the Generator

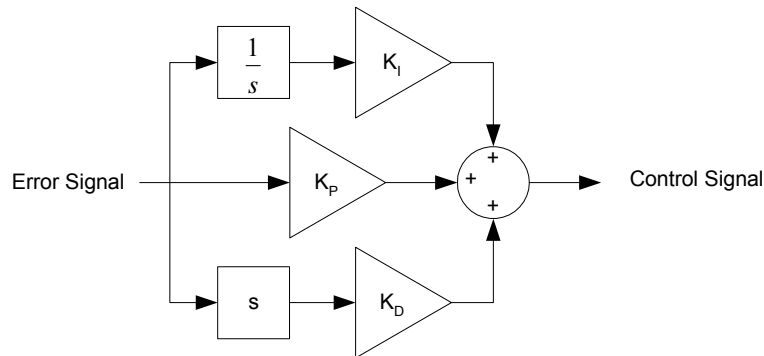


Figure 8: A PID controller. The ' $1/s$ ' term takes the integral of the error signal

7a) Sketch a block diagram of the complete generator set model with a **proportional-integral-derivative** controller and the transfer functions for the engine and generator. Use the generator transfer function model from Figure 3. **$J = 1000$ and $B_G = 100$** . Leave B_L as a variable.

Lesson 9: PID Controllers

7b) Implement a proportional-integral-derivative controller in the SIMULINK model. Adjust the gain terms K_P , K_D , and K_I to improve the generator set's response (both at startup and under load). Record the gain values and steady state speed. **Print the "Generator Speed" plot for your notes.** The "Generator Speed" plot should be scaled to show the whole response.

Annotate the plot with:

- The point when full load is applied.
- The steady state generator speed under full load.
- The settling time of the load transient.

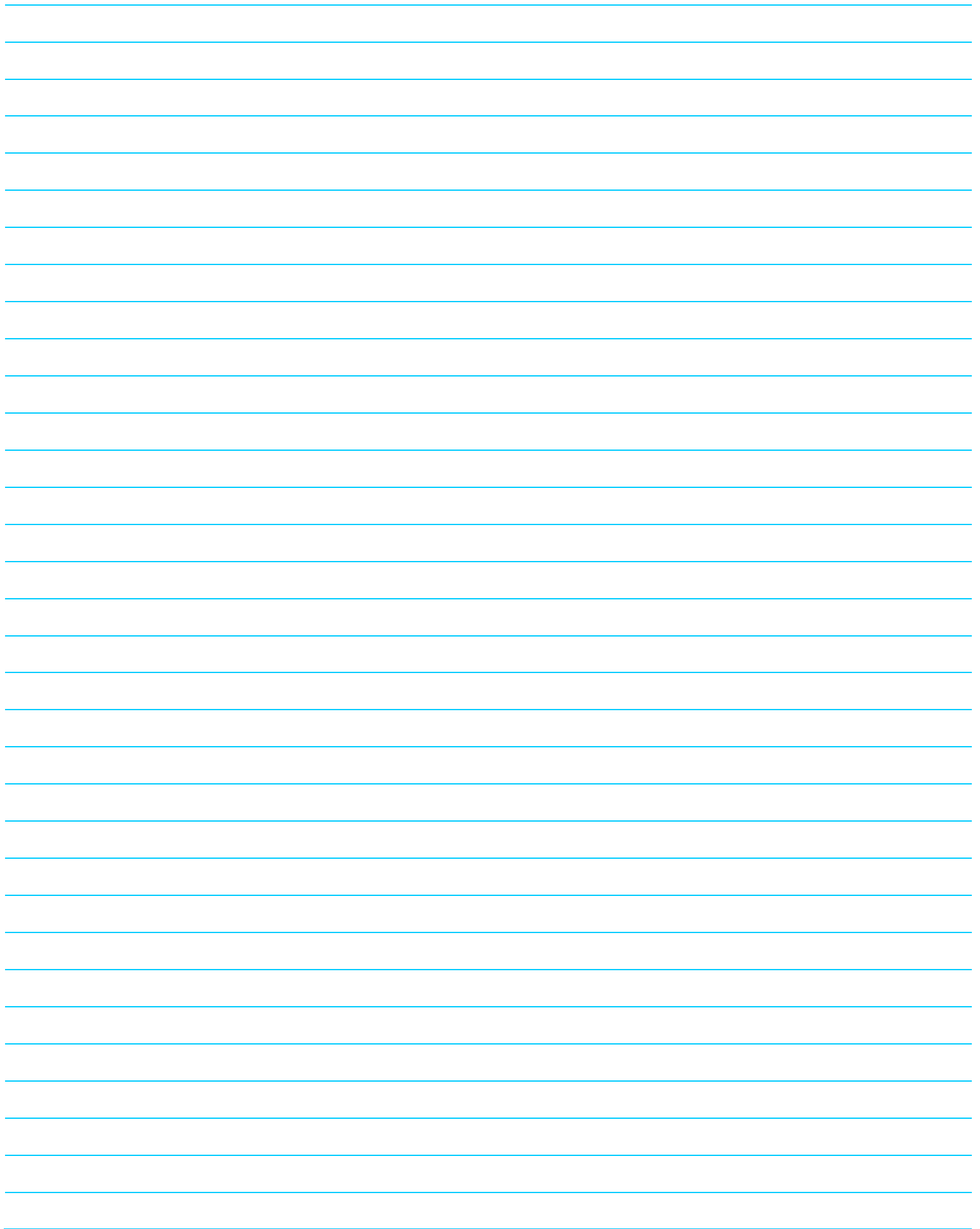
$K_P =$	$K_D =$	$K_I =$
Final RPM =		

7c) Now vary the value of the integral gain and describe how the response changes. Set the following: **Simulation stop time = 30 seconds, $K_P = 500$, and $K_D = 600$.** How does increasing the integral gain affect the system's response?

K_I value	Settling Time for Full Load Transient
0	
100	
200	
500	
1000	

7d) Increase the integral gain term to a very large value (greater than 50,000). **Set the simulation stop time to 15 seconds.** Run the simulation. **Print the "Generator Speed" plot for your notes.** What kind of response is occurring in the first 10 seconds? What is the maximum RPM reached by the generator? Use the answer to question 6c to explain why this happens for large values of K_I .

Response: (0 to 10 sec)	Maximum RPM:
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Lesson 10: PID Control and Discrete Sampling

Part I: PID Control of the Cruise Missile's Speed

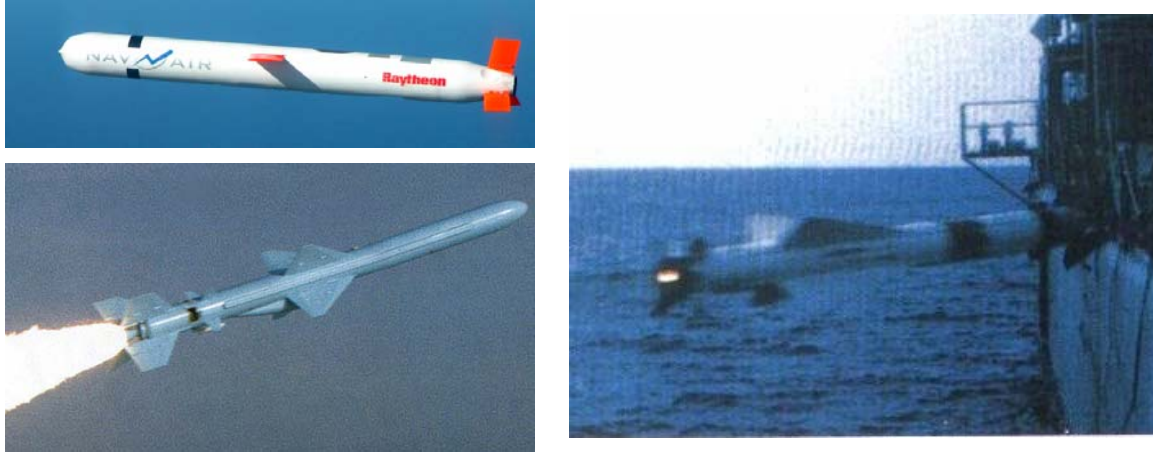


Figure 1: The Tomahawk Land Attack Missile (TLAM), upper left. A Chinese YJ86LG, lower left. An Exocet impacting a target, right.

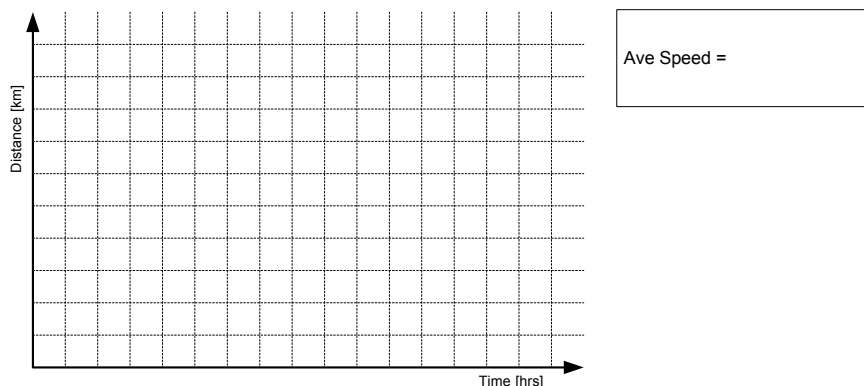
This lesson continues the exploration of a PID controller. For maximum strike effectiveness, a cruise missile's "time on target" should be as close to the programmed value as possible. The PID controller will regulate the speed of a cruise missile to try to achieve the correct time on target. Typical parameters for a Tomahawk Land Attack Missile (TLAM) are listed in Table 1.

Simulation TLAM Parameters	
Maximum Speed	880 km/hr
Minimum Speed	400 km/hr
Maximum Range	1100 km

Table 1: TLAM Parameters

The mission profile can be graphed on a plot of distance vs. time. Initially ($t=0$) the missile is at the launch platform ($d=0$).

1a) What is the average speed required to reach a target 1000 km away in exactly 2 hours? Plot this mission profile on the axes below. How is speed depicted on this plot?



Lesson 10: PID Control and Discrete Sampling

A simple model of the TLAM’s speed control is illustrated in Figure 2. The input to this model is the missile’s commanded position (distance from launch site) as a function of time. The cruise missile’s transfer function describes the missile’s ability to speed up and slow down. The system monitors actual cruise missile position (again distance from the launch point) and compares it with commanded position to generate the error signal.

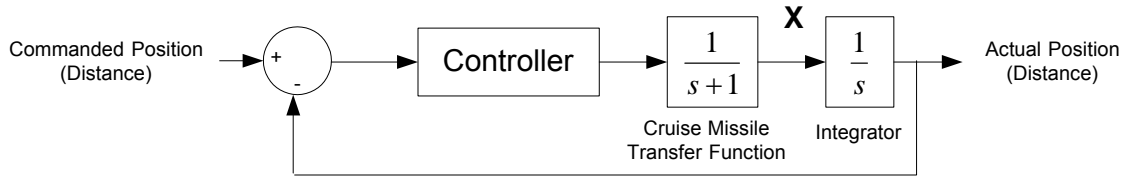


Figure 2: The block diagram for the cruise missile’s speed control system.

1b) Label the error signal and the control signal on the above block diagram.

1c) What signal is represented by the “x” in the above block diagram? Hint: The integral of “x” is the cruise missile’s position.

1d) If the missile arrives at its target on time, what is the value of the error signal?

Error Signal =

Preparing the Model for Use

Download and open the file **cruise_missile.mdl** in SIMULINK. Open the controller subsystem and implement a PID controller. See Lesson 9 if you need help remembering how a PID controller is implemented in SIMULINK.

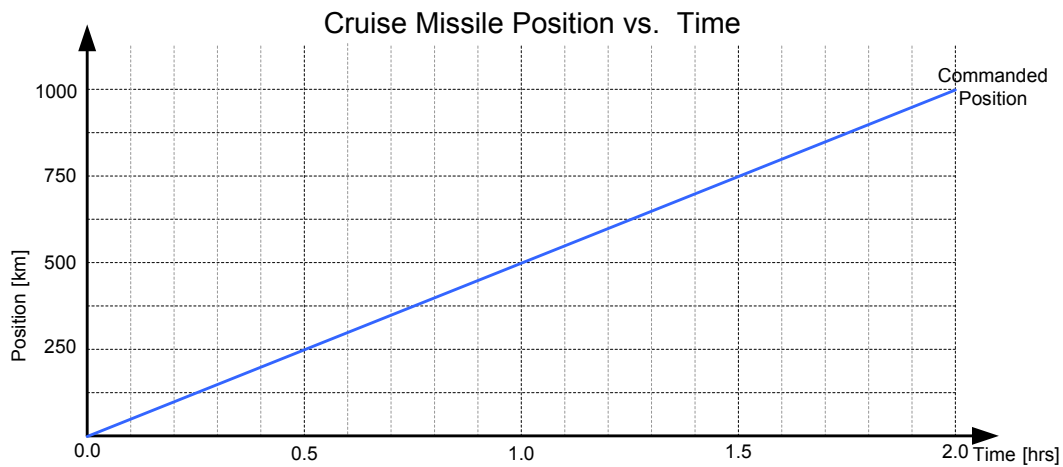
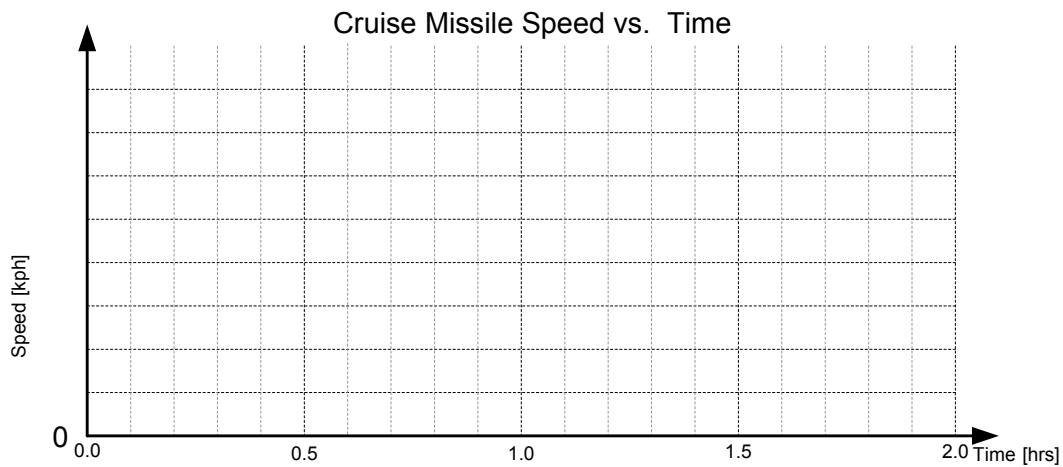
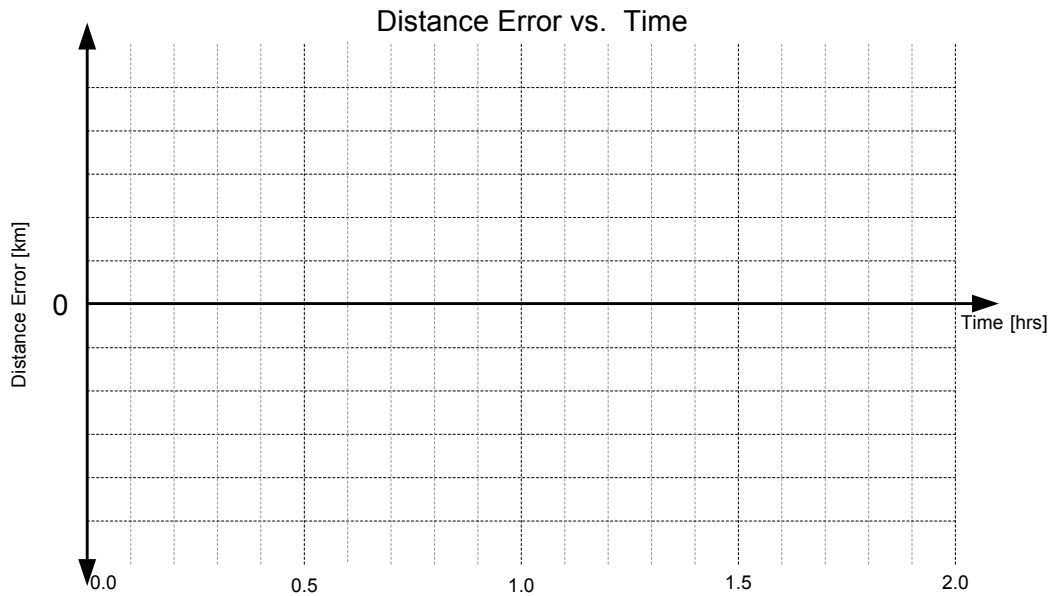
Set or check the following model parameters prior to running the simulation:

- The commanded position is specified using a ramp function. **Ensure the slope of the ramp is the average speed calculated in question 1a.**
- In the configuration parameters window (Ctrl-E) set the stop time of the simulation to 2 and the “Relative Tolerance” to 1e-6.
- In the PID controller set the gains as follows: **$K_P=100$, $K_D=0$, and $K_I=0$.**¹

¹ By setting the K_D and K_I gains to 0, the PID controller becomes a simple proportional controller.

Lesson 10: PID Control and Discrete Sampling

2a) Run the simulation and sketch the plots of “CM speed”, “Distance Error”, and “Distance” on the axes below. Label the maximum, minimum, and final values on the “CM Speed” and “Distance Error” plots.



2b) After launch, what is the cruise missile’s minimum and maximum speed?

Min Speed [kph]:	Max Speed [kph]:
------------------	------------------

2c) The speeds predicted in 2b are a problem. Why? Hint: See Table 1.

Limiting the Cruise Missile Speed

To limit the cruise missile's speed to a range of realistic values insert a "Saturation" block in the SIMULINK model between the cruise missile's transfer function and the integrator. Set the upper and lower limits of the saturation block to the cruise missile's minimum and maximum speeds specified in Table 1.

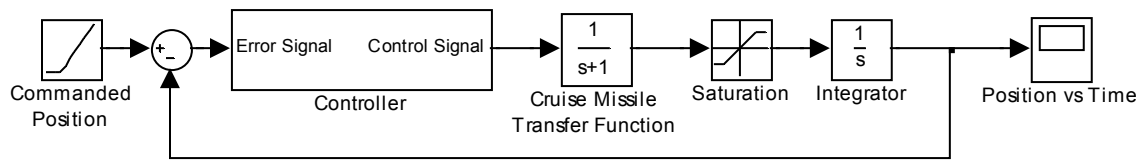
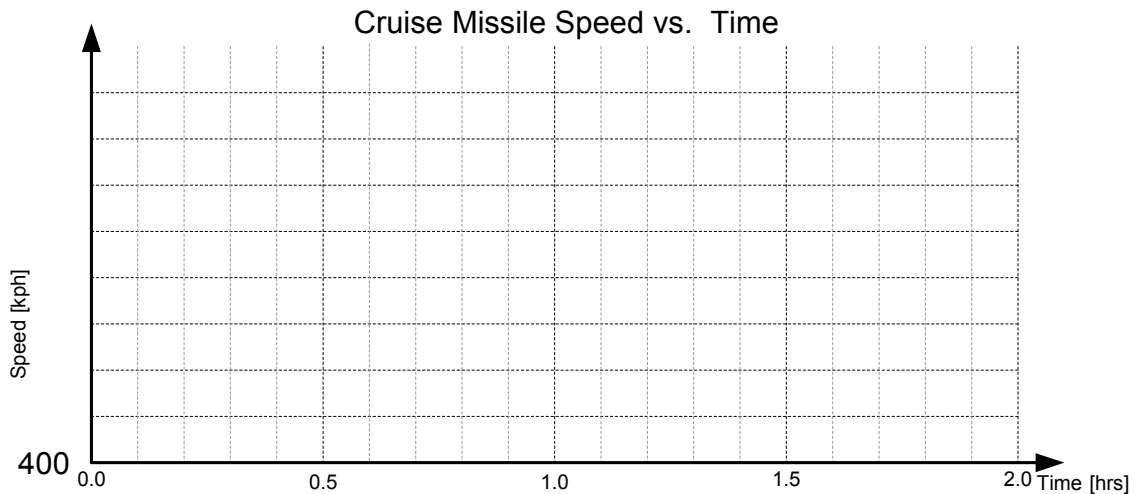


Figure 3: The SIMULINK model with a saturation block to limit the cruise missile's speed. Many of the plotting and display elements in the model have been omitted for clarity.

2d) Run the simulation again and sketch the Cruise Missile Speed plot on the axes below.



2e) The maximum cruise missile speeds predicted by the model without the saturation block (question 2b) is greater than the upper limit for the saturation block. Looking at the cruise missile speed plot in question 2d you can clearly see the saturation block limiting the lower speed of the missile. However, the missile speed never gets fast enough to be limited by the saturation block's upper limit. Why?

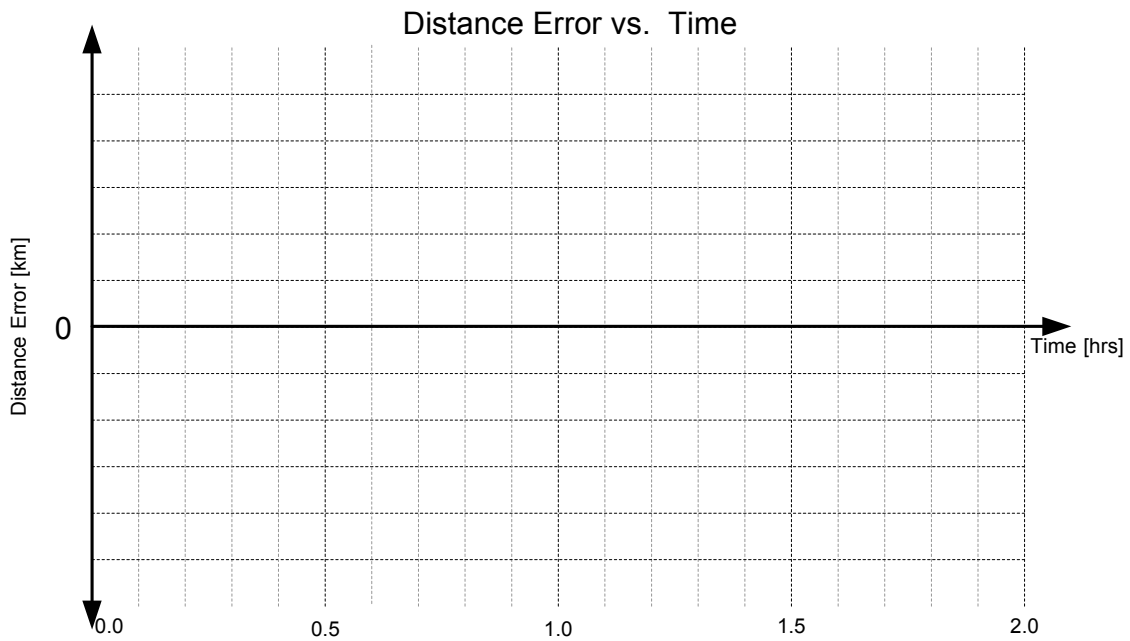
Adjusting the Transient Response

3a) Which gain in the PID controller adjusts the system's transient response and only the system's transient response?

3b) What gain value will make the response critically damped? Increase the gain (from question 3a) until there is no overshoot in the cruise missile's speed plot. Record this value to the nearest 10.

3c) Record the controller's gain values and the distance error at 1 and 2 hours of flight. Sketch a plot of the distance error for these gain values.

K_P :	K_D :	K_I :	Distance Error at 1 hour [km]	Distance Error at 2 hours [km]
---------	---------	---------	-------------------------------	--------------------------------

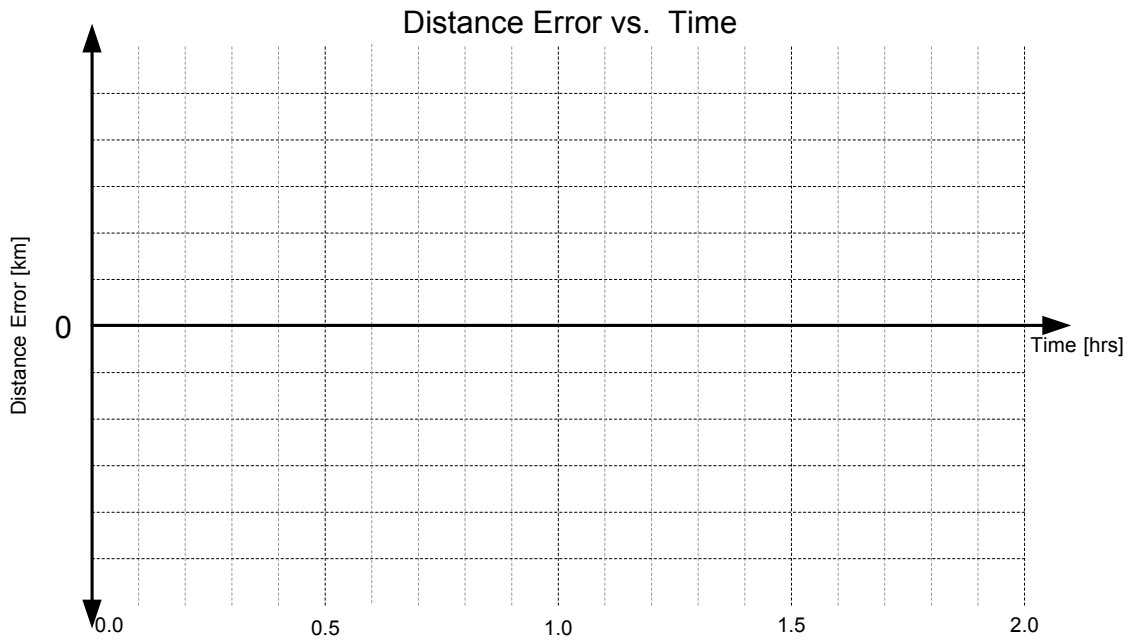


Adjusting the Steady State Response

4a) Which gain term will adjust the system's steady state value and only the system's steady state value?

4b) Find the minimum value for this gain which minimizes the cruise missile's distance error at 2 hrs. Record the gain value and the distance errors at 1 and 2 hours of flight. Sketch a plot of the distance error for these gain values.

K_P :	K_D :	K_I :	Distance Error at 1 hour [km]	Distance Error at 2 hours [km]
---------	---------	---------	-------------------------------	--------------------------------



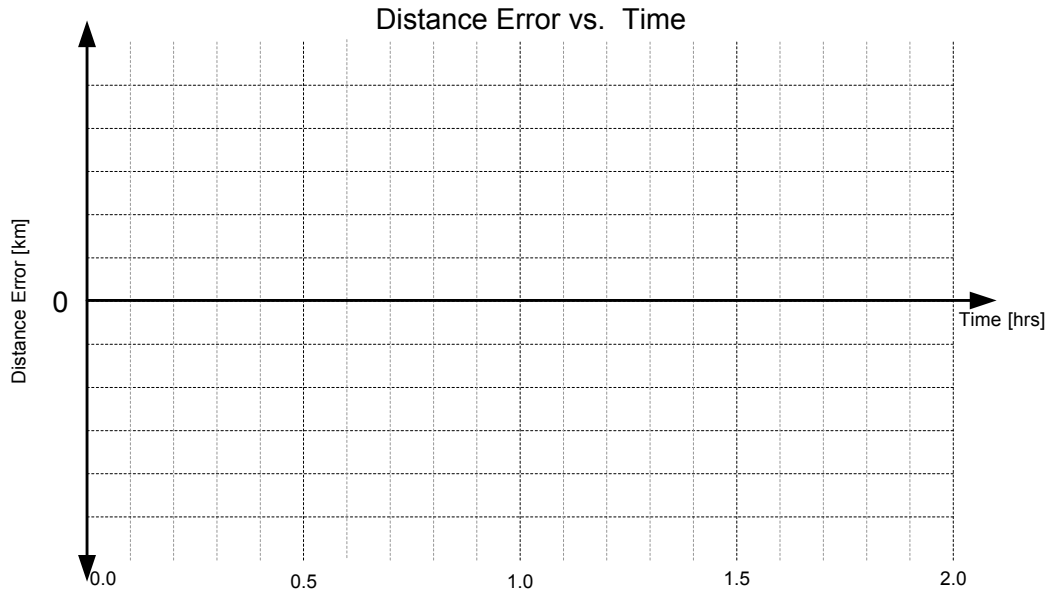
4c) Now try to also reduce the cruise missile's distance error at 1 hour into flight. What happens to the distance error at 2 hours? Why?

Final Controller Tuning

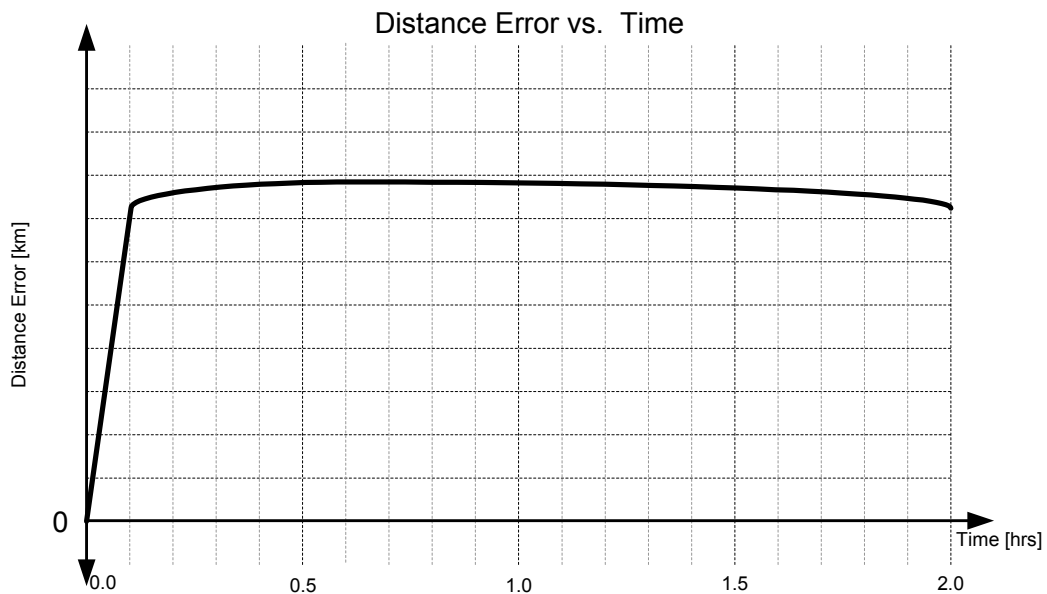
5a) Now adjust all three gains so the cruise missile's distance error is less than 1 km for most of the flight. Additionally, all gain values should be less than 1000. Record your gain values below.

K_P :	K_D :	K_I :	Distance Error at 1 hour [km]	Distance Error at 2 hours [km]
---------	---------	---------	-------------------------------	--------------------------------

5b) Sketch a plot of the distance error for these gain values.



5c) The plot below illustrates cruise missiles distance error over its flight. Illustrate how **increasing** each of the three gains (K_P , K_D , and K_I) change the shape of this plot.



Part II: Discrete Sampling

This section of the lesson will explore the differences between continuous and discrete sampling. In **continuous sampling** the feedback signal to the controller is always available and up to date at any and all instances of time. In **discrete sampling** the feedback signal to the controller is only updated periodically.

6a) Consider this simple example. When you are walking with your eyes open, you are receiving continuous feedback about your position with respect to objects around you. Now try walking around the room with your eyes closed, only open them once per second. How easy is it to walk quickly and avoid obstacles? How easy is it to run? Now try walking around the room again, this time only peek every 10 seconds. With your eyes closed longer, does this make it easier to move fast and avoid obstacles? Why or why not?

When you are walking with your eyes open your brain is a feedback system with continuous sampling. When walking with your eyes shut and only peeking occasionally, your brain is a feedback system with discrete sampling.

Discrete Sampling and the Cruise Missile

The controller in the cruise missile needs a measurement of the missile's current position to accurately fly the mission profile. A simple method of providing this position feedback is with a GPS receiver. However, most GPS receivers only provide position updates periodically (i.e. not continuously). This can be especially true in inclement weather or if obstacles are between the receiver and one or more of the satellites.

6b) How will the cruise missile's performance change as the amount of time between position updates **increases**?

Discrete Sampling in SIMULINK.

Discrete sampling can be modeled in SIMULINK using the Zero-Order Hold block.² The Zero-Order Hold block measures its input periodically. Its output, until the next sample time, is the value of the previously measured input.

Figure 4 shows the model (a) and results (b) of a ramp signal processed by a Zero-Order Hold block with a sampling period of 1.

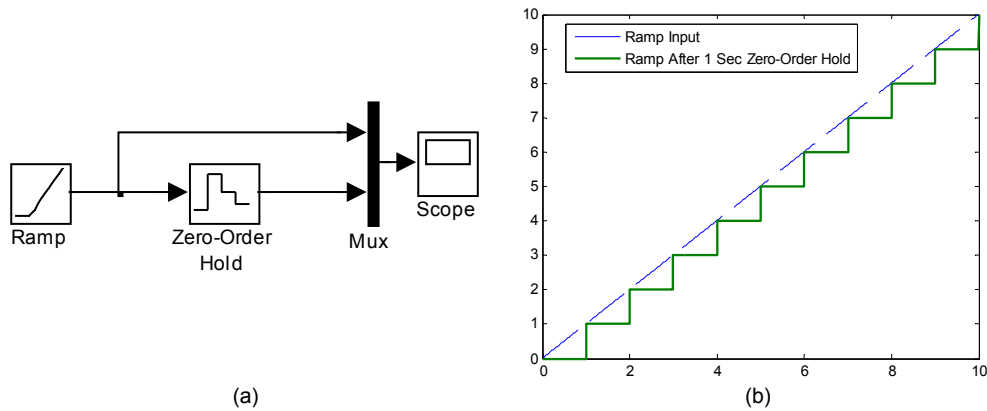


Figure 4: A Zero-Order Hold block processing a ramp signal (a) and the plot of the results (b).

Adding Discrete Sampling to the Cruise Missile’s Position Measurement

7a) Set the controller’s gains to $K_P=100$, $K_I=100$, and $K_D=70$. Run the simulation (without a Zero-Order Hold block in the model) to establish a baseline for performance. Record the Maximum distance error, and the distance errors at 1 and 2 hours of flight. Sketch a plot of the distance error for these gain values.

Maximum Distance Error [km]	Distance Error at 1 hour [km]	Distance Error at 2 hours [km]
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² The Zero-Order Hold block is located in the “Discrete functions” section of the SIMULINK Library.

Lesson 10: PID Control and Discrete Sampling

Add a Zero-Order Hold block to the feedback loop of the cruise missile model as depicted in Figure 5. Double click on the block and set the sample time to 1/60. This sets the sample period to once per minute because the simulation time scale is in hours. You will need to flip the block around to make it fit into the model.

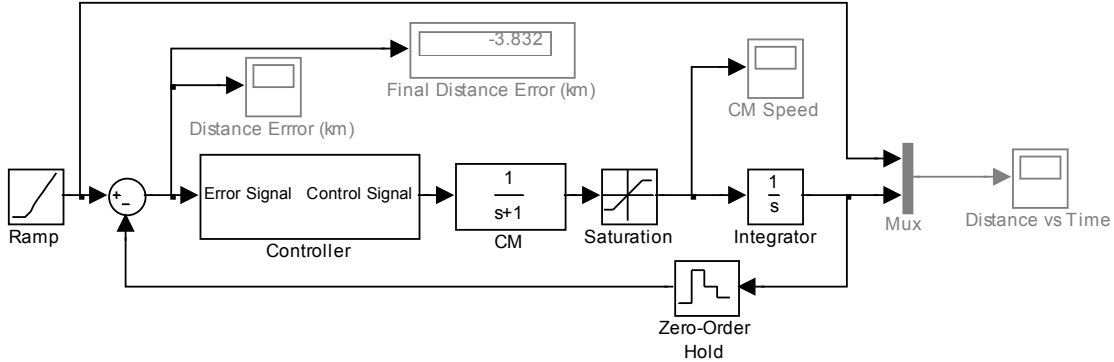
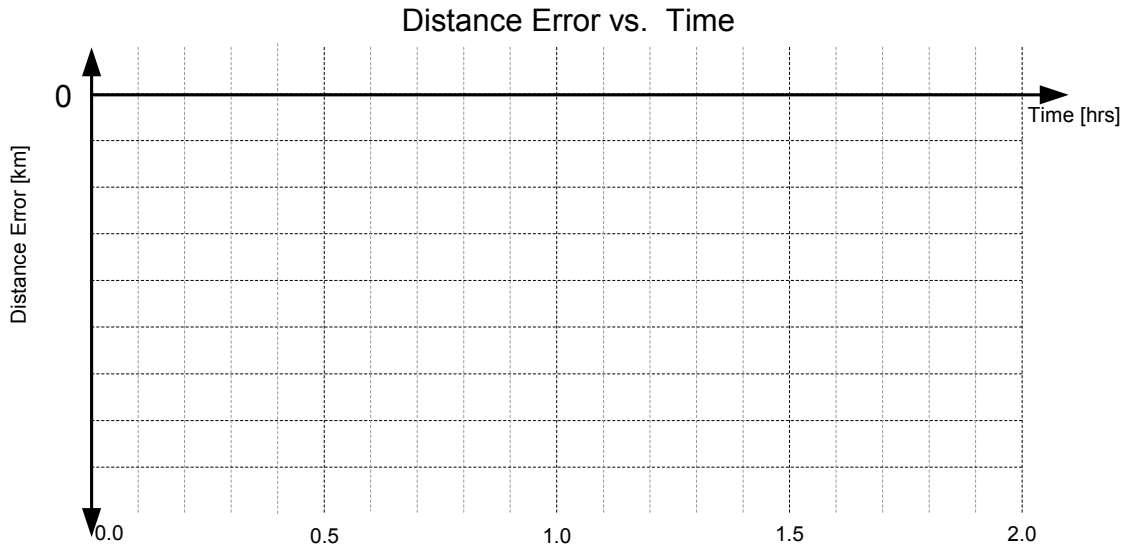


Figure 5: The cruise missile model with a Zero-Order Hold block providing discrete sampling of the missile's position down track. Model elements in gray provide display functions only and do not affect the model's dynamics.

7b) Run the simulation. Record the maximum distance error and the distance errors at 1 and 2 hours of flight. Sketch a plot of the distance error for these gain values and sampling period.

Maximum Distance Error [km]	Distance Error at 1 hour [km]	Distance Error at 2 hours [km]



7c) What happened?!?

Lesson 10: PID Control and Discrete Sampling

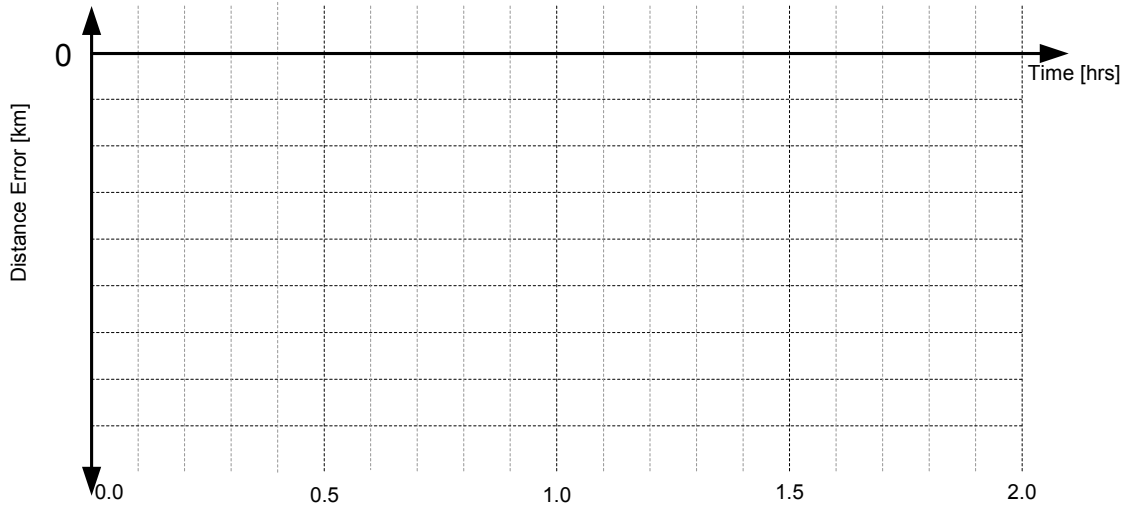
7d) Zoom in on the a section of the Distance Error plot to see its shape clearly. **Label this plot with your name and print it for your notes.** Describe the line plotted.

7e) In the example of someone walking with their eyes shut and only peeking occasionally, how does one improve the system's response?

7f) Increase the sampling rate of the Zero-Order Hold block to once per second (1/3600 hrs), and rerun the simulation. Record the maximum distance error and the distance errors at 1 and 2 hours of flight. Sketch the distance error plot on the axes below.

Maximum Distance Error [km]	Distance Error at 1 hour [km]	Distance Error at 2 hours [km]
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Distance Error vs. Time



7g) How much did increasing the sampling time improve the system's response?

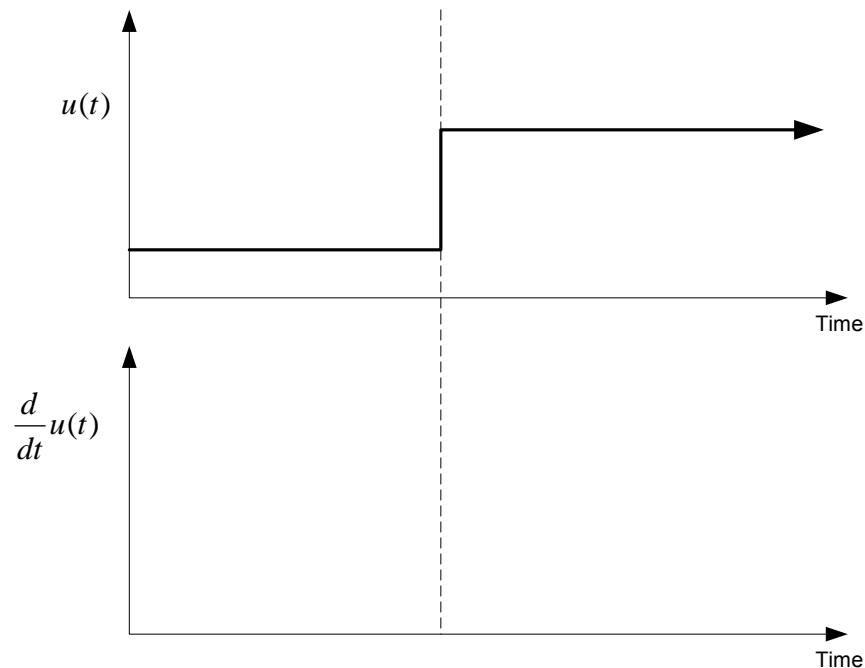
The “Other” Discrete Sampling Problem

Question 7 illustrates that even with the GPS providing an updated position to the cruise missile for each second of a two hour flight (7,200 position updates), the missile’s performance will still be much worse than if its position has been updated continuously.

This is because sampling rate is not the only concern when it comes to discrete control systems. Question 8 explores this “other” problem with discrete sampling.

8a) What is the derivative of a **continuously sampled** ramp signal?

8b) Plot the derivative of the step function on the axes below.



8c) Check your answer with SIMULINK. Open a new model window and build the model depicted in Figure 6. Use the default values for the step function (Step Time = 1, Initial Value = 0, Final Value = 1).

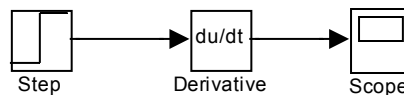



Figure 6: A SIMULINK model to determine the derivative of a step function.

Record the maximum value for the derivative. The autoscale button  on the plot is helpful in determining this value.

The Derivative of a Step
Function’s Maximum Value

Lesson 10: PID Control and Discrete Sampling

Now build the SIMULINK model depicted in Figure 7 to find the derivative of a discretely sampled ramp signal. Use a Mux block to plot three signals on the same plot. Double click on the Mux block to add another input port.

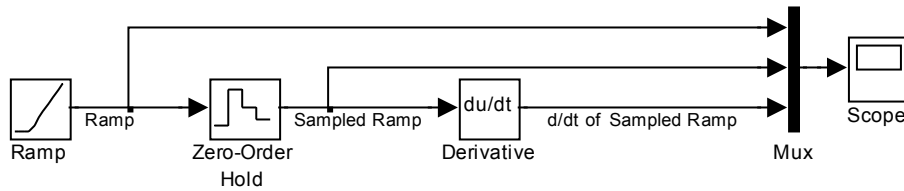


Figure 7: A SIMULINK model to find the derivative of a discretely sampled ramp signal.

8d) Sketch the derivative of a **discretely sampled** ramp signal?



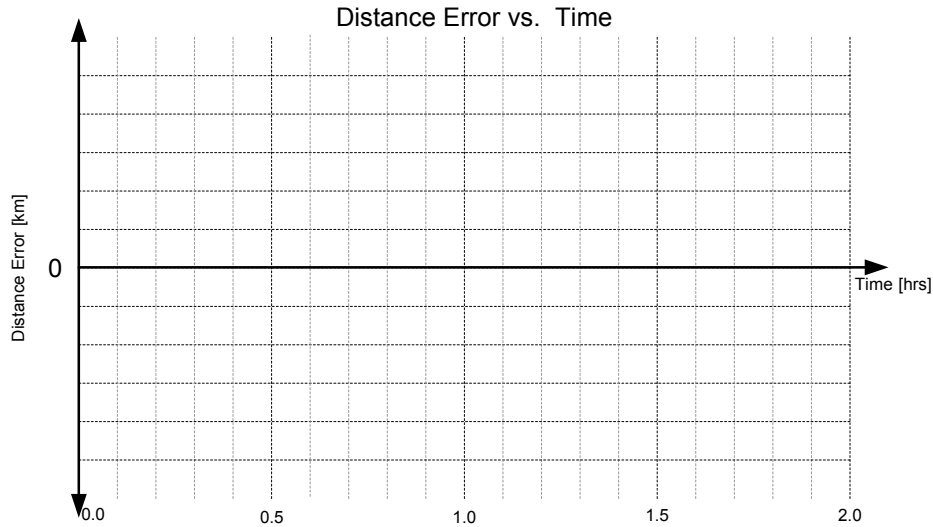
8e) Which part of the cruise missile's controller is causing the missile's performance to suffer because of the discrete position sampling?

8f) List two methods of fixing the controller to improve its performance for a discretely sampled feedback signal. Using a continuous feedback signal is **not** an option!

Lesson 10: PID Control and Discrete Sampling

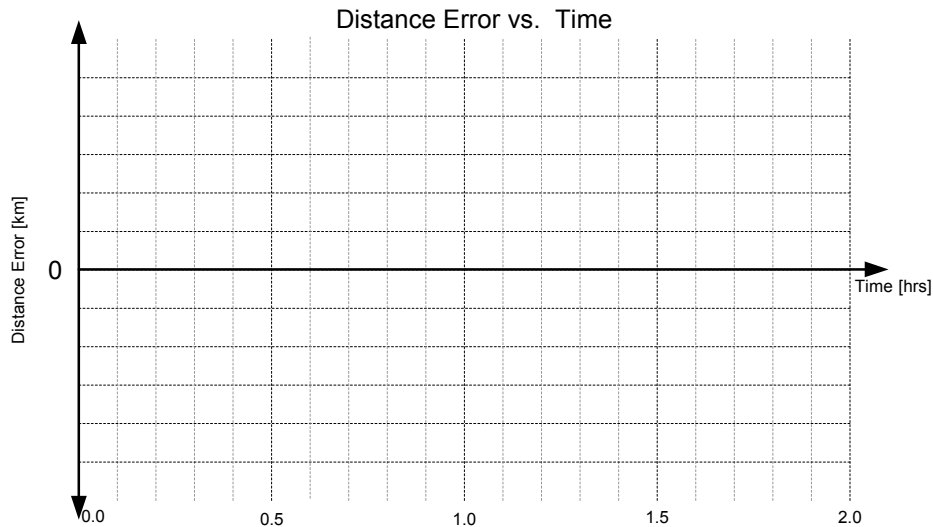
9a) Set the controller's gains to $K_P=100$, $K_I=100$, and $K_D=10$. Set the sampling period of the Zero-Order Hold block to once per minute (1/60 hrs). Run the simulation again. Record the maximum distance error and the distance errors at 1 and 2 hours of flight. Sketch the distance error plot for these gain values and sampling period.

Maximum Distance Error [km]	Distance Error at 1 hour [km]	Distance Error at 2 hours [km]
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9b) Remove the controller's derivative term completely (set $K_D = 0$). Run the simulation again. Record the maximum distance error and the distance errors at 1 and 2 hours of flight. Sketch the distance error plot for these gain values.

Maximum Distance Error [km]	Distance Error at 1 hour [km]	Distance Error at 2 hours [km]
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9c) How much did lowing K_D to 10 improve the response? Lowing K_D to 0?

Lesson 10: PID Control and Discrete Sampling

Removing the derivative term of the controller is one way of improving the response of a system with discrete feedback. However, this causes the controller to be much less effective at controlling the transient response of the system.

We would like to leave the derivative term in the controller to control the transient response of the system. The discontinuities from sampling every minute cause the derivative term in the controller to give erroneous control signals. The magnitude of these erroneous control signals are very large and dwarf the actual control signals.

“Smoothing out” the position feedback signal will remove the discontinuities causing the problems. A first order transfer function can smooth out the feedback signal. This filtering transfer function should have a DC gain of 1 and be of the form

$$G = \frac{\alpha}{s + \alpha}.$$

Where α is selected to smooth out the feedback signal. Insert a first order transfer function into the cruise missile's SIMULINK model. Add another Scope block and a Mux block to plot the feedback signals entering and exiting the filtering transfer function. Figure 8 depicts the model with these elements added to it.

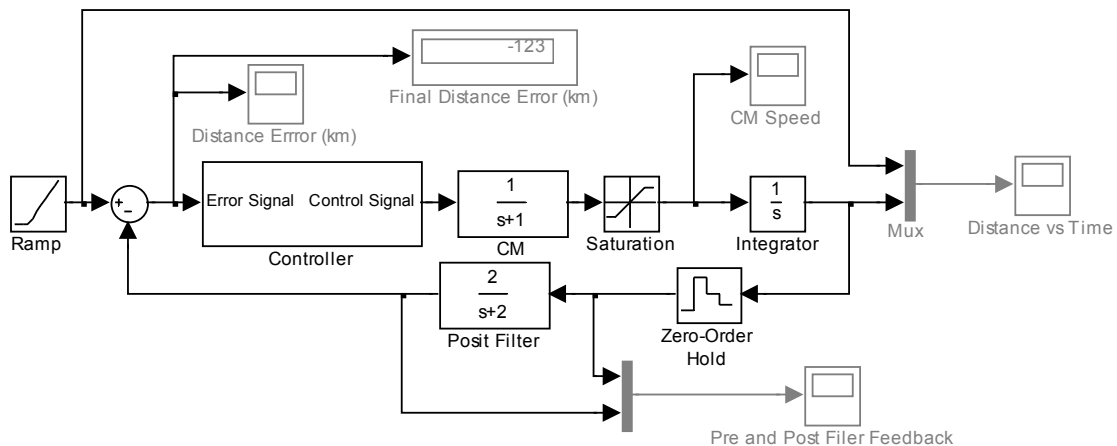


Figure 8: The cruise missile SIMULINK model with a 1st order transfer function to smooth the discretely sampled position feedback signal. The Mux and Scope blocks display the input to and output from the “Position Filter” block.

10a) **Set the sample time of the Zero-Order Hold block to once per minute (1/60 hrs).** In Figure 8, $\alpha = 2$. This value of α does not allow the filter's output to accurately track its input. Run the simulation and adjust α so the position filter's output closely tracks its input while at the same time smoothing its output. Record your value of α below.

α

Set all the controller gains to 100. Set the sample time of the Zero-Order Hold block to once per minute (1/60 hrs).

10b) Run the simulation again and **print the "Cruise Missile Speed" and "Distance Error" plots for your notes.** Label the plots with the gain values and sampling period. How is the cruise missile's speed changing over the duration of the flight?

10c) Now set the sample time of the Zero-Order Hold block to once per second (1/3600 hrs). Run the simulation again and **print the "Cruise Missile Speed" and "Distance Error" plots for your notes.** Label the plots with the gain values and sampling period. How is the cruise missile's speed changing over the duration of the flight?

10d) Has the faster sampling rate improved the cruise missile's response?

10e) In this question, adjusting the sampling rate does affect the cruise missile's response. In question 7, adjusting the sampling rate had little affect on the cruise missile's response. Explain this difference in system behavior?

Hardware Implementation of PID Speed Control

This lesson continues the exploration of PID controllers. Figure 1 shows the cart that will be used for this lab. A digital PID controller is used to regulate the cart's speed.

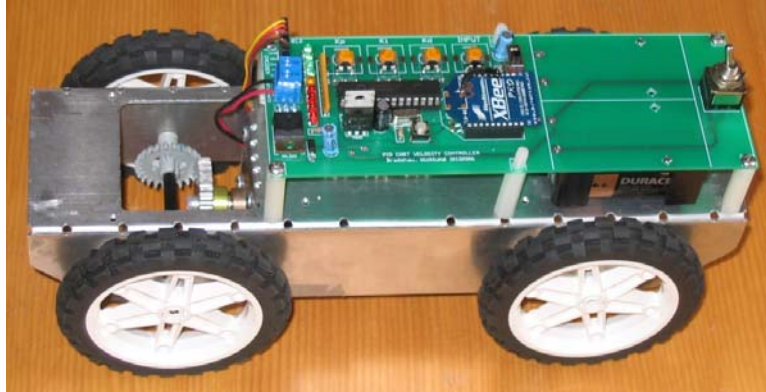


Figure 1: The PID controlled cart.

Cart Overview

The **INPUT** adjustment sets the desired speed for the cart. The **K_p**, **K_i**, and **K_d** adjustments set the controller's proportional, integral, and derivative gains. The **program mode switch** determines which of the two possible program modes (green or red) the cart will operate in. The **start button** sets the cart in motion and resets the cart at the end of a run.

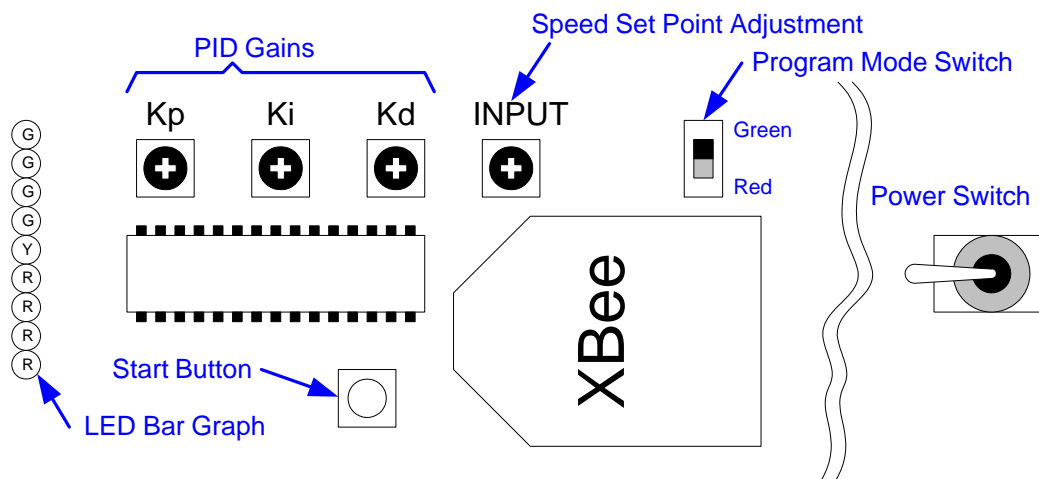


Figure 2: The cart's controls.

The primary function of the **LED bar graph** is to display the sign and magnitude of the error signal when the cart is in motion. The yellow LED indicates the error is zero (the cart is traveling at the desired speed). Red LEDs indicate the cart is traveling slower than the desired speed. Green LEDs indicate the cart is traveling faster than the desired speed. The farther from the yellow LED, the greater the magnitude of the error.

Additionally, the **LED bar graph** has three secondary functions:

- It confirms which mode the cart is in by blinking either the green or the red LEDs.
- When the speed set point is adjusted with the **INPUT** potentiometer, the bar graph indicates the specified speed. The yellow LED corresponds to a desired speed set point of 0.5 m/s. Red LEDs indicate a slower set point and green LEDs indicates a faster set point.
- When the **start button** is pressed, the bar graph counts down until the cart starts moving.

The cart's speed is measured using an encoder on the motor shaft.

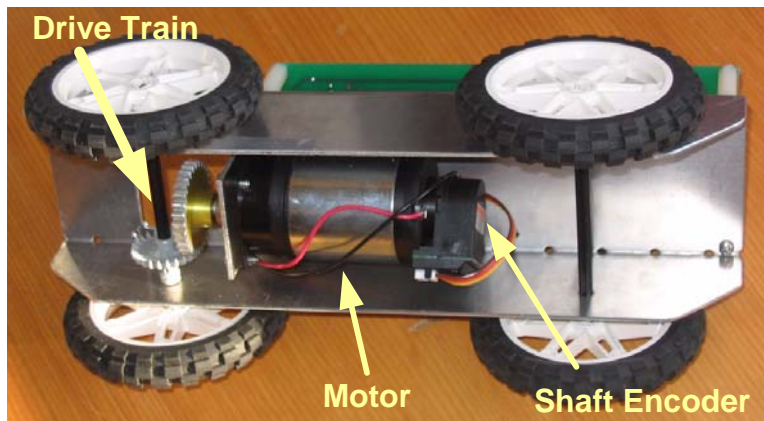


Figure 3: The underside of the cart showing the drive train, motor, and encoder.

Cart Operation

1. Set the **program mode switch** to the desired mode (green or red).
2. Turn the cart on. The LEDs will blink indicating the selected mode.
3. Set the desired cart speed with the **INPUT** adjustment. The LEDs will indicate desired speed. The yellow LED indicates a desired cart speed of 0.5 meters per second. Red is slower and green is faster.
4. Adjust the PID gains (**K_p**, **K_i**, and **K_d**) as desired.

Clockwise (CW) is the maximum gain value.

Counter clockwise (CCW) is a gain of zero.

5. Press the **start button**. The LEDs will count down and the cart will start moving.
6. At the end of the run the cart will stop and the yellow LED will blink.
7. Press the **start button**. The program mode will be displayed and then the speed set point and PID gains may be adjusted in preparation for the next run.

Before we turn the cart loose in the hallway, let's become familiar with the response of its PID controller.

Lesson 11: Hardware Implementation of PID Speed Control

Proportional Gain Effects

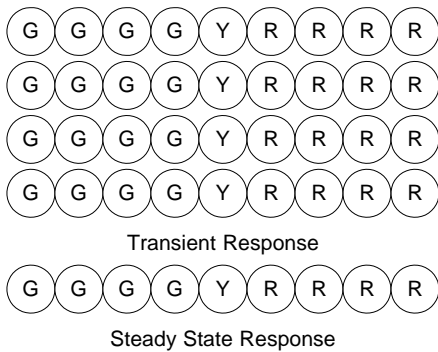
1a) Configure the controller as specified in the table below. Hold the cart in your hand and press the start button to run the cart. Which LEDs are lit? Does the steady state error indicate faster or slower than the desired speed?



Program	Green
Kp	Full CCW (zero)
Ki	Full CCW(zero)
Kd	Full CCW (zero)
INPUT	Yellow LED only

1b) Increase the proportional gain by **slightly** turning the potentiometer clockwise and run the cart again. Which LEDs are lit? What has happened to the steady state error? While the cart is running in your hand, you can vary the proportional gain to see the effect on steady state error.

1c) Maximizing the proportional gain can minimize steady state error at the expense of the transient response. Configure the controller as specified in the table below. Hold the cart in your hand and run it. Describe the transient response indicated by the LEDs. What is the steady state error?



Program	Green
Kp	Full CW (maximum)
Ki	Full CCW (zero)
Kd	Full CCW (zero)
INPUT	Yellow LED only

Integral Gain Effects

Recall that the integral term affects the steady state response.

2a) Establish a baseline response by configuring the cart as specified in the table below. Hold the cart in your hand while running it. Note the steady state error. This is the response with NO integral term ($K_i = 0$).



Program	Green
K_p	Mid-Position
K_i	Full CCW (zero)
K_d	Full CCW (zero)
INPUT	Yellow LED only

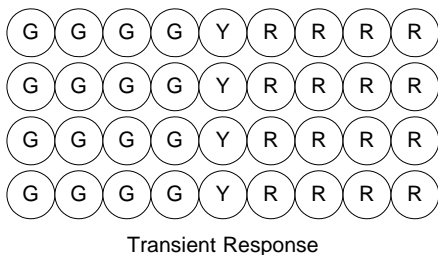
2b) Now **very slightly** increase the integral gain (turn **K_i** clockwise). What is the steady state error? How long does it take the controller to reach the steady state condition?



2c) Increase the integral gain some more. Do **not** maximize the integral gain (fully clockwise). How does the system's response change? Repeat this several times varying the amount of integral gain.



2d) Now let's examine what happens when the integral gain becomes too large. Configure the cart as specified in the table below, and hold it in your hand while running it. What is the steady state error? What happens to the transient response? **Why?**



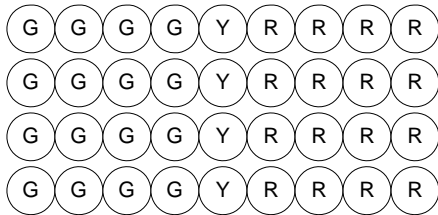
Program	Green
K_p	Full CCW(zero)
K_i	Full CW (Maximum)
K_d	Full CCW (zero)
INPUT	Yellow LED only



Derivative Gain Effects

Recall that the derivative gain affects only the transient response.

3a) Configure the cart to produce an under damped response by maximizing the proportional gain and setting the derivative and integral gains to zero specified in the table below. Hold it in your hand while running it. What is the largest oscillation of the error signal? What is the steady state error?



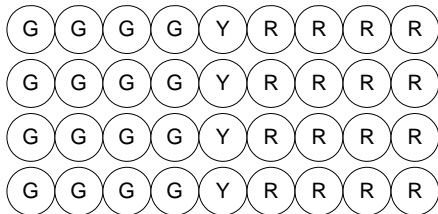
Transient Response



Steady State Response

Program	Green
Kp	Full CW (Maximum)
Ki	Full CCW (zero)
Kd	Full CCW (zero)
INPUT	Yellow LED only

3b) Increase the derivative gain and run the cart again. Describe the transient response.



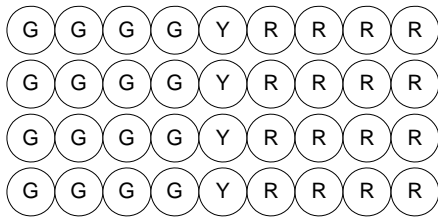
Transient Response

Discrete Sampling Problems

4a) In Lesson 10 we explored the “other” problem with discrete sampling. What was this “other” problem?

4b) This controller samples the shaft encoder at 50Hz to calculate the cart’s speed error. In the green program mode, a filter is applied to the error signal to smooth it out. In the red program mode, no filtering is applied to the error signal. Predict how the cart will respond in the red program mode.

4c) Test your prediction by running the cart with no filtering applied to the error signal (red program mode). Configure the cart as described in the table below. Describe the transient response.



Transient Response

Program	Red
Kp	Full CW (Maximum)
Ki	Full CCW (Zero)
Kd	Full CW (Maximum)
INPUT	Yellow LED only

4d) Run the cart again. This time, change the program mode back and forth during the run. How does the response change?

4e) Vary the proportional and derivative gains while running the cart in the red program mode. Which values of these gains make the response worse?

Lesson 11: Hardware Implementation of PID Speed Control

Running the Cart

5a) Configure the cart as follows and run it up the big ramp. Note the steady state error.



Program	Green
Kp	Mid-Position
Ki	Full CCW (zero)
Kd	Full CCW (zero)
INPUT	Yellow LED only

5b) If Kp is increased what will happen to the steady state error? Will it be eliminated?

5c) Maximize Kp and run the cart up the big ramp. Note the steady state error?



Program	Green
Kp	Full CW (maximum)
Ki	Full CCW (zero)
Kd	Full CCW (zero)
INPUT	Yellow LED only

5d) What will happen to the steady state error if a very slight amount of integral gain is used?

5e) Apply a **very slight** amount of integral gain and run the cart up the big ramp. Note the steady state error?



Program	Green
Kp	Full CW (maximum)
Ki	Almost Full CCW (but not quite)
Kd	Full CCW (zero)
INPUT	Yellow LED only

5f) Now run the cart **down** the big ramp and note the steady state error.



Program	Green
Kp	Full CW (maximum)
Ki	Almost Full CCW (but not quite)
Kd	Full CCW (zero)
INPUT	Yellow LED only

5g) Adjust the gains to remove the steady state error going **down** the big ramp.

Lesson 11: Hardware Implementation of PID Speed Control

5h) Were you able to remove the steady state error in question 5e? Why or why not?

5i) Add more friction to the cart by dragging a mass behind the cart as it runs down the ramp. Can you remove the steady state error now? Why or why not?

The cart is programmed to run a little over 5 meters. The goal is to have the cart run the 5 meter course in exactly 10 seconds. There are two 5 meter courses, one carpeted and one not carpeted. Run the cart several times on each. Adjust the gains to get the cart to run the course in exactly 10 seconds. You may use different gain settings for each course.

The cart should be run in the green program with the desired speed set to 0.5 meters per second (the yellow LED).

5j) Record your attempts in the table below.

Course	Speed	Kp	Ki	Kd	Time	Comments

5k) For which course was it easier to have the cart meet the goal time? Why?