Introduction

This handout will give you information on some basic principles when choosing what motor to use for what application in your machine.

1) All the calculations here assume that the motor is running at a constant velocity. The acceleration is very short and therefore can be ignored here. DO ALL CALCULATIONS AT THE STEADY STATE, NOT THE ACCELERATION STATE.

2) Your wheels or winch will dissipate NO power if there is no slippage between the wheel and the ground (or winch and cable).

Torque

For the case of a wheel or winch the force is always tangent.

\[ T = F \cdot r \]

Figure above shows the basic torque equation as a function of force, angle, and radius. The units of torque are Nm or ft-lbs.

Motors Torque & Speed

In our steady state model of motors there are two parameters that we are interested in.

The Stall Torque (Ts):

The minimum torque needed to completely stop the motor shaft from rotating (this is
The No Load Speed (\(W_n\)):
The rotational speed of the motor shaft when there is NO torque being applied to it.
Units are RPM or rad/s

These two parameters allow us to define the following two equations:

\[
\tau = \tau_s - \frac{\tau_s}{\omega_n} \omega
\]

\[
\omega = (\tau_s - \tau) \frac{\omega_n}{\tau_s}
\]

The plot of either one of these equations is called a torque speed curve. For this model of an electric motor the torque speed curve is linear. The plot below is for the Bosch Motor.

The Bosch motors have a no load speed of 95 RPM and a stall Torque of 0.6 Nm.

Thus if we know the torque applied to the motor shaft we can find how fast the motor will rotate. Or if we know how fast we want the motor to spin, we can find how much torque should be applied.

**Motor Power**

Recall that power equals

\[
P = \tau \omega
\]
By multiplying either equation (4) by \( W \) or equation (5) by \( T \) we get equations that describe Power as functions of either torque or velocity.

\[
P(\omega) = -\frac{\tau_s}{\omega_n} \omega^2 + \tau_s \omega
\]

\[
P(\tau) = -\frac{\omega_n}{\tau_s} \tau^2 + \omega_n \tau
\]

Both of these equations are quadratics. They are plotted below.
Notice that there is a maximum power for a given range of speed and torque. For the motor to do the most it should be operating at torque and speed of maximum power.

The optimum torque and speed are half the Ts and Wn respectively.

The maximum power available is the TsoWn/4

mtrpwr.m plots the torque speed curve and power curves for given values of stall torque and no load speed.

mtrpwr(Ts,Wn)

»help mtrpwr

[Tmax,Wmax] = MTRPWR(Ts,Wn) Given the Stall Torque [Ts] in [N-m] and No Load Speed [Wn] in [rpm] returns the Torque of maximum power output [Tmax] in [Nm] and the rotational speed of maximum power out [Wmax]
\( T = F r r \) is the torque exerted by the motor to overcome the friction.

The wheel with spin at speed \( w \)

\[
\omega = \left( \tau_s - \tau \right) \frac{\omega_n}{\tau_s}
\]

\[
\omega = \left( \tau_s - F_{fr} \right) \frac{\omega_n}{\tau_s}
\]

\[
\omega = \left( \tau_s - \mu N \right) \frac{\omega_n}{\tau_s}
\]

Equation 8 is only true as long as

\[
r\mu N \geq \tau_s
\]

When \( r\mu N = \tau_s \) the motor stalls and everything stops.

**Moving Vehicles**

Lets now look at the case of a moving vehicle.

There are three things to remember:

1. If the wheels roll without slipping no power is dissipated by friction between the wheels and the road.
2. If there are no external forces on the vehicle it will travel at a constant velocity.
3. Work in the steady state!

**Case 1:**

A free rolling vehicle with no external forces and no weight.

This is the no load case. Assuming no internal friction the wheel will turn at \( \omega_n \). The speed of the vehicle will be \( r\omega_n \).

**Case 2:**
Free rolling with no external forces and a lot of weight.

Your intuition is correct that this will go slower. The change is due to internal friction. As the weight goes up, the friction in the system goes up. The torque from the motor goes up to balance this. As some point the internal friction will stall the motor and stop the entire machine.

As the load goes up, the steady state velocity will decrease linearly.

**Case 3:**

This case is when the machine is pushing something- like another machine or a ball. To push the object requires some force \( F_p \).

\[ F_l r = \tau \]

To overcome the load force the motors need to exert a torque to overcome it.

The resultant rotational speed will be:

\[ \omega = \left( F_l r - \tau_s \right) \frac{\omega_n}{\tau_s} \]

One of two failure modes will occur, this depends on the coefficient of friction, the normal force \( N \), the stall torque, the radius \( r \), etc.
Designing and building winches is an area where students tend to get into trouble. You MUST do some calculations to verify that the motor you are using can lift your anticipated load.
<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
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<tbody>
<tr>
<td><strong>Winch Calculations</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>By: Roger Cortesi</td>
<td></td>
<td></td>
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<tr>
<td>Modified on: 21 JAN 98</td>
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<tr>
<td>gravity [m/sec^2]:</td>
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<td>Motor no load speed, Wm [RPM]:</td>
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<td>Wn [rad/s]</td>
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<td>Motor Stall Torque, Ts, [Nm]:</td>
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<td>Length of arm, La [inches]:</td>
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<td>Length of winch connection, Lw [inches]:</td>
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<td>Lw [m]:</td>
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<td>Radius of Winch, r [inches]:</td>
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<td>mass at tip of arm [kg]:</td>
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<td>Torque needed by motor [Nm]:</td>
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<td>Speed of Motor lifting arm, Wl [rad/s]:</td>
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<td>Wl [RPM]</td>
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<td>Linear speed at tip [m/s]:</td>
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*winch.xls* calculates the data needed to size winch drums, arm lengths, and motors ratings.

[Download winch.xls]

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