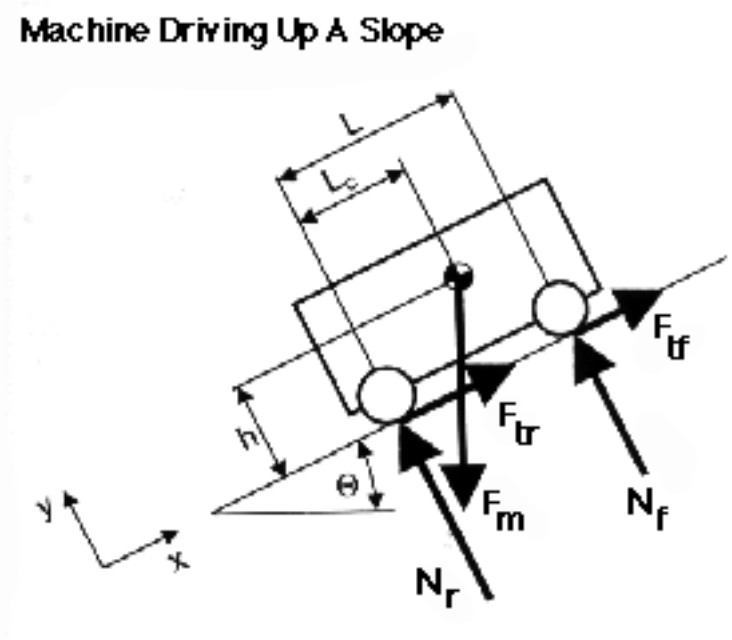
Climbing an Incline Plane

Introduction:

This handout will examine the physics behind wheeled vehicles climbing slopes.



Fm = mg	(gravitational force of the machine)
Nf	(normal force of both front wheels)
Nr	(normal force of both rear wheels)
Ftf = Nf μ	(tractional force of both front wheels, caused by static friction)
Ftr = Nr μ	(tractional force of both rear wheels, caused by static friction)
L	(distance between the front and rear wheels)
Lc	(distance between the rear wheels and the center of mass)
h	(height of the center of mass)
Ø	(angle of incline plane)

$$\sum F_y = 0 : N_r + N_f - F_M \cdot \cos\Theta = 0$$

$$\sum M_r = 0 : F_M \cdot h \cdot \sin\Theta + N_f \cdot L - F_M \cdot L_c \cdot \cos\Theta = 0$$

$$\rightarrow N_f = \frac{F_M \cdot L_c \cdot \cos\Theta - F_M \cdot h \cdot \sin\Theta}{L}$$

$$\rightarrow N_r = \frac{F_M \cdot (L - L_c) \cdot \cos\Theta - F_M \cdot h \cdot \sin\Theta}{L}$$

Tipping of the Machine occurs at h=hmax when Nf=0:

$$N_f = 0 : \frac{F_M \cdot L_c \cdot \cos\Theta - F_M \cdot h_{max} \cdot \sin\Theta}{L} = 0$$

$$\rightarrow h_{max} = \frac{L_c}{\tan\Theta}$$

What can be done to prevent the machine from tipping?

- moving the center of gravity farther from the rear wheels
- decreasing the slope angle

Four Wheel Drive:

The necessary static coefficient of friction (μ_{4WD}) for a four wheel drive vehicle:

$$N_r \cdot \mu_{4WD} + N_f \cdot \mu_{4WD} \geq F_M \cdot \sin\Theta$$

$$\rightarrow \mu_{4WD} \geq \tan\Theta$$

What can be changed to allow for a lower coefficient of friction?

$$N_f \cdot \mu_{FWD} \geq F_M \cdot \sin\Theta$$

$$\rightarrow \mu_{FWD} \geq \frac{1}{\frac{L_c}{L \cdot \tan\Theta} - \frac{h}{L}}$$

What can be changed to allow for a lower coefficient of friction?

- decrease the anlge of the incline
- move the center of gravity closer to the front wheels
- shortening the distance between the front and rear wheels
- lowering the height of the center of gravity

Rear Wheel Drive:

The necessary static coefficient of friction (μRWD) for a rear wheel drive vehicle:

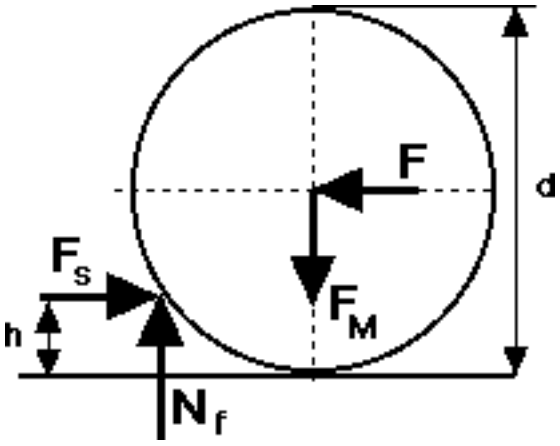
$$N_r \cdot \mu_{RWD} \geq F_M \cdot \sin\Theta$$

$$\rightarrow \mu_{RWD} \geq \frac{1}{\frac{L - L_c}{L \cdot \tan\Theta} - \frac{h}{L}}$$

What can be changed to allow for a lower coefficient of friction?

- decrease the angle of the incline
- move the center of gravity closer to the rear wheels
- increasing the height of the center of gravity

A Wheel Driving Up a Step



$$\sum M_f = 0 : F \cdot \left(\frac{d}{2} - h\right) - F_M \cdot \sqrt{d \cdot h - h^2} = 0$$

$$\rightarrow F = F_M \frac{\sqrt{d \cdot h - h^2}}{\frac{d}{2} - h}$$

$0 < h < d/2$

This site is maintained by rcortesi@mit.edu;
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