Stiffness and Bending

Introduction

One very common problem that students have in 2.007 is not making their arm or structures stiff enough. This is a problem because the arms and structures usually need to move or support things. A lack of stiffness is very common cause of machine unreliability.

Remember from 2.001 that the following factors need to be known to calculate the stiffness of something.

The Young's Modulus [E]:
This is a material property that measures the stress/strain.

The Cross-Sectional Inertia [I]:
This is determined by the cross sectional geometry of the arm.

The Loading Configuration:
This gives your equation to calculate the the deflection. Typical configurations and their equations are listed below.

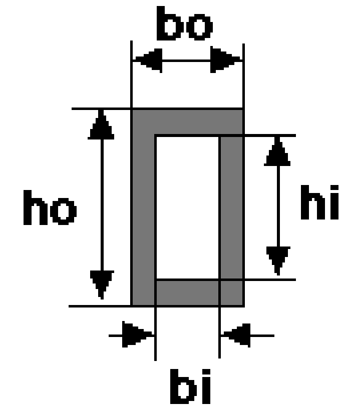
Young's Modulus

Material	Young Modulus E		Shear Modulus G		Poisson's Ratio v
	ksi	GPa	ksi	GPa	
Aluminum (pure)	10,000	70	3,800	26	0.33
Aluminum alloys					
6061-T6	10,000	70	3,800	26	0.33
7075-T6	10,400	72	3,900	27	0.33
Steel	29,000	190-210	11,300	75-80	0.27-0.30
Delrin		3.1			0.35

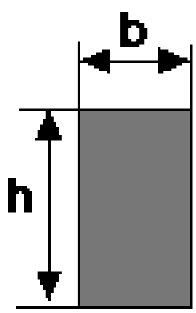
Moments of Inertia

Return to Top of Page

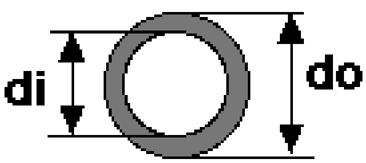
Cross Section	Inertia



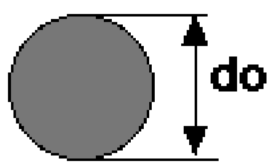
$$I_x = \frac{b_o \cdot h_o^3 - b_i \cdot h_i^3}{12}$$



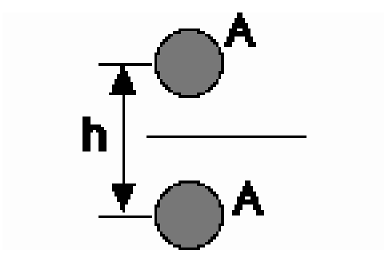
$$I_x = \frac{b \cdot h^3}{12}$$



$$I_x = \frac{\pi \cdot (d_o^4 - d_i^4)}{64}$$

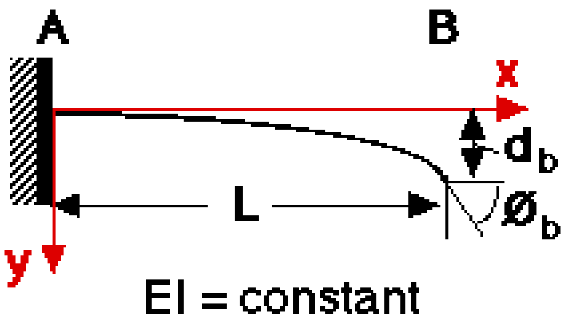


$$I_x = \frac{\pi \cdot d^4}{64}$$



$$I_x = 2A\left(\frac{h}{2}\right)^2$$

This is an approximation of a simple truss, ingoring the cross members. Both upper and lower members have the same area (A).

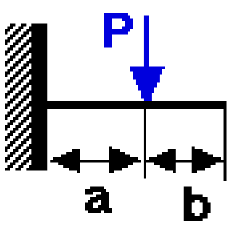


v = deflection in y direction

v' = dv/dx = slope of deflection curve

db = v(L) = deflection at right end of beam

Theta b = v'(L) = angle at right end of beam



0 < X < A

$$v = \frac{Px^2}{6EI}(3a - x)$$

$$v' = \frac{Px}{2EI}(2a - x)$$

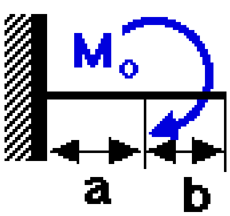
A < X < L

$$v = \frac{Pa^2}{6EI}(3x - a)$$

$$v' = \frac{Pa^2}{2EI}$$

$$\delta_b = \frac{Pa^2}{6EI}(3L - a)$$

$$\theta_b = \frac{Pa^2}{2EI}$$



$$v = \frac{M_o x^2}{2EI}$$

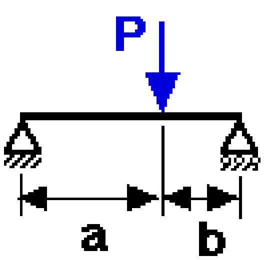
$$v' = \frac{M_o x}{EI}$$

$$v = \frac{M_o a}{2EI}(2x - a)$$

$$v' = \frac{M_o a}{EI}$$

$$\delta_b = \frac{M_o a}{2EI}(2L - a)$$

$$\theta_b = \frac{M_o a}{EI}$$



$$v = \frac{Pbx}{6LEI}(L^2 - b^2 - x^2)$$

$$v' = \frac{Pb}{6LEI}(L^2 - b^2 - 3x^2)$$

Substitute a for b and (L-x) for x, to calc v and v' for a < x < L

$$\theta_a = \frac{Pab(L + b)}{6LEI}$$

$$\theta_b = \frac{Pab(L + a)}{6LEI}$$

Tools for Evaluating Deflection

Return to Top of Page

There are some tools to help you calculation the amount of deflection in your structures. If you forget how to

bending.xls

	A	B	C	D
1	Simple and Cantilevered Beam Calculations			
2	By : Roger Cortesi			
3	Late Modified on 4 FEB 98			
4	**Modify Numbers in Bold Only**			
5	To mach variable names see bending handout			
6				
7	**Material Properties**			
8	**Young's Modulus of Material, E [GPa]:**	**200**	E [Pa]:	2E+11
9				
10	**Cross Section Configurations**			
11	Rectagular			
12	**Ho [mm]:**	**4**	Ho [m]:	0.004
13	**Hi [mm]:**	**0**	Hi [m]:	0.000
14	**bo [mm]:**	**4**	bo [m]:	0.004
15	**bi [mm]:**	**0**	bi [m]:	0.000
16	Ix [m^4]:	2.13E-11		
17				
18	Circular			
19	**Do [mm]:**	**4**	Do [m]:	0.004
20	**Di [mm]:**	**0**	Di [m]:	0.000
21	Ix [m^4]:	1.26E-11		
22				
23	**Bending Configurations**			
24	**Length, L [mm]:**	**100**	L [m]:	0.100
25	**Applied Load, P [N]:**	**400**	P [lbs]:	89.9
26	**Load Distance, A [mm]:**	**75**	A [m]:	0.075
27	**Cross Sectional Inertia, I [m^4]:**	**2.13E-11**		
28	**Point of Interest, X [mm]:**	**75**	X [m]:	0.075
29				
30	Cantilevered Beam			
31	Deflection at X, v [m]:	0.013	v [mm]:	13.2
32				
33	Simply Supported Beam			
34	b = (L-a) [m]:	0.025		
35	(L-x) [m]:	0.025		
36	Deflection at X, v [m]:	0.001	v [mm]:	1.1
37				

Sheet1 / Sheet2 / Sheet3

recinert.m and **cirinert.m** calculate the cross sectional inertia for rectangular and circular cross sections respectively. all units must be consistent.

RECINERT(A,B,C,D) and **CIRINERT(A,B)**

```
»help recinert

  RECINERT(A,B,C,D) calculates the cross sectional moment of interia for
```

```
  a rectangular cross section
  A = outer width, B = outer height
  C = inner width, and D = inner height

  To evalute a filled rectangle set C and D = 0
```

```
»help cirinert

  CIRINERT(A,B) calculates the cross sectional moment of interia for
  a pipe cross section
  A = outer diameter, B = inner diameter

  To evalutate a filled circle set B = 0
```

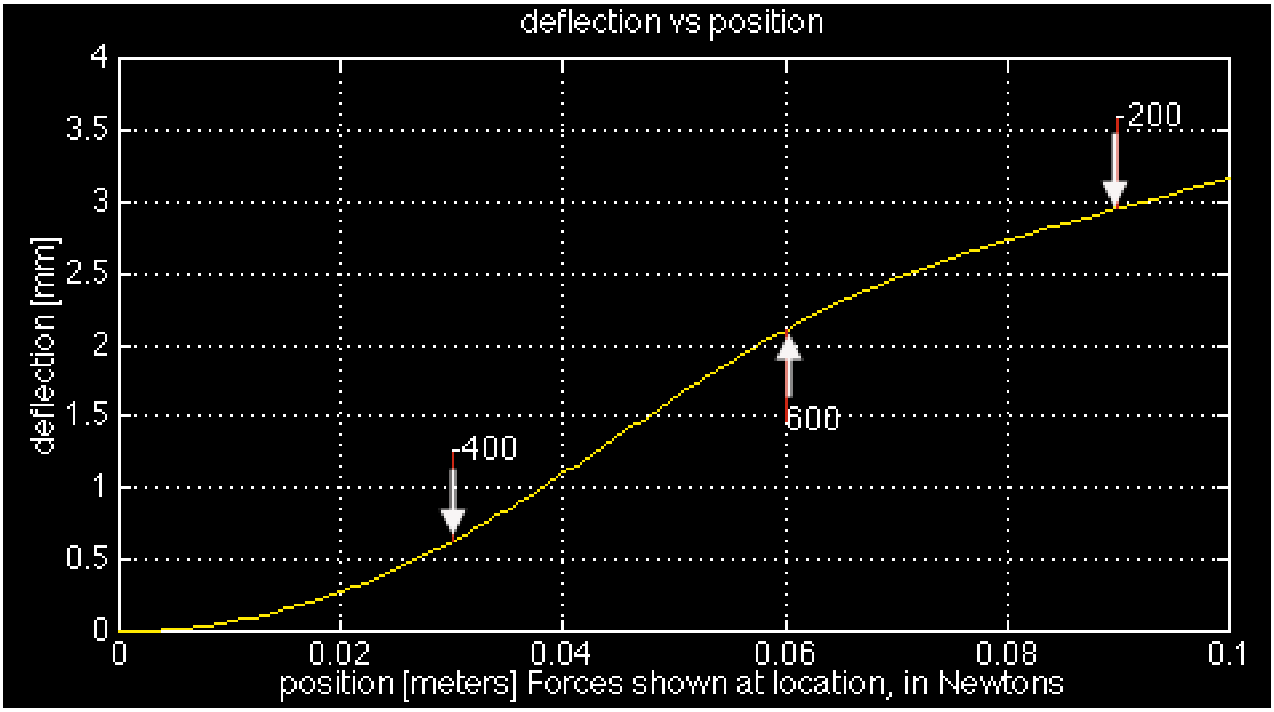
cantbeam.m shows the deflection of a cantilevered beam loaded from 1 or more points.

CANTBEAM(P,a,L,E,I,incr)

```
»help cantbeam

  [def] = CANTBEAM(P,a,L,E,I,incr) returns arrays with
  the deflection [def].  [def] is also plotted.


        P is the force applied to the beam in NEWTONS
        a is the distance from the left end that the force is applied in METERS
        L is the length of the beam in METERS
        E is the young's modulus of the material in Pa
        I is the cross sectional inertia in METER^4
        icnr is the number of increments to sample along the beam

  Multiple forces can be entered in P, however, a must be the same length
  to give a position for each force.  P can be positive or negative.

  all values of a may not be greater then L
  E, I, a, incr, and L must be greater then 0

  def is in meters (plotted in mm)
```

Below is a sample plot from cantbeam.m for the following data:

```
L = 0.10 m
I = 2.1333e-11 m^4   (4mm by 4mm shaft)
E = 200E+9 Pa  (steel)
incr = 100
P = [-400 600 -200] N
a = [0.03 0.06 0.09] m
```



On the deflection plot the red lines are at the point of force application and they are "pushing" the beam (e.i. a positive force will have its red line below the beam "pushing" up). The values of each force is displayed at the end of its force line.

The deflection is plotted in **mm** but the array returned for [def] is in **meters**!

[simpbeam.m]

shows the deflection of a simple beam supported

at either

end, loaded from 1 or more points.

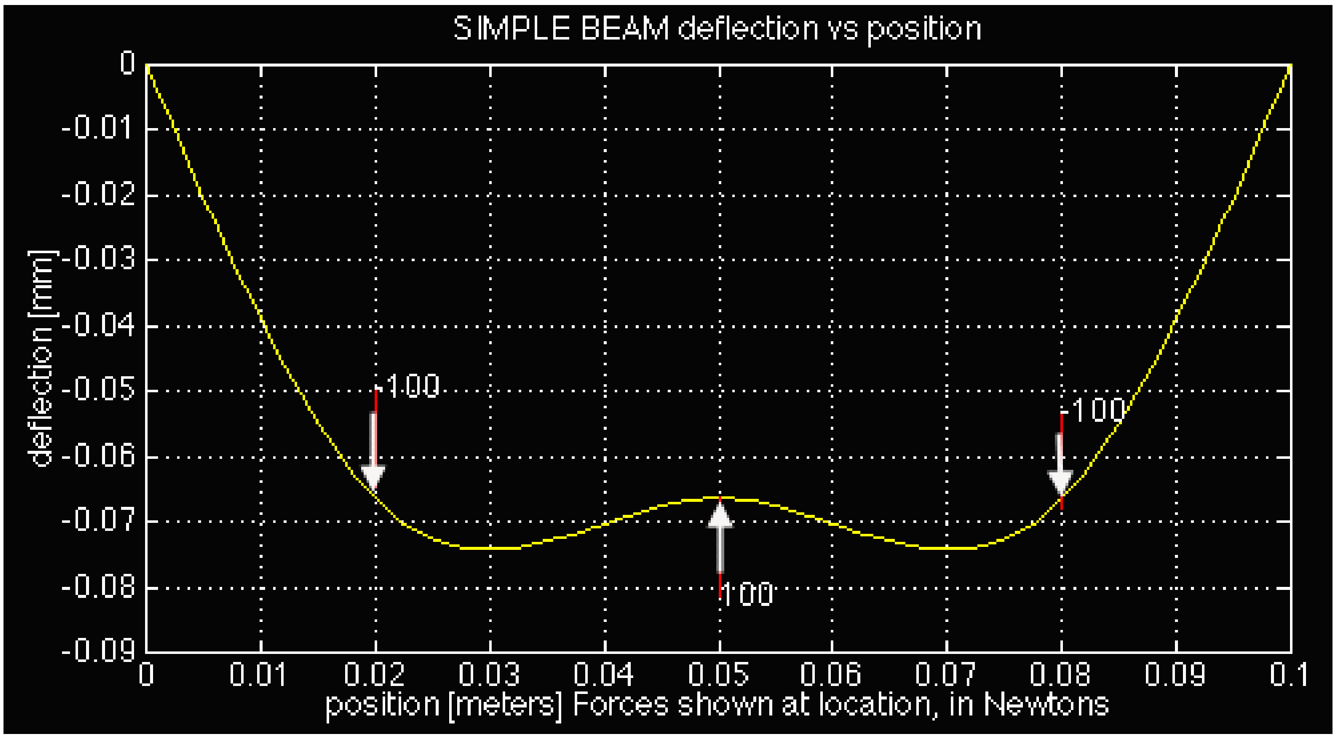
SIMPBEAM(P,a,L,E,I,incr)

```
»help simpbeam

  [def] = SIMPBEAM(P,a,L,E,I,incr) returns an array with
  the deflection [def].  [def] is also plotted.
```

```
Multiple forces can be entered in P, however, a must be the same length
to give a position for each force.  P can be positive or negative.

all values of a may not be greater then L
E, I, a, incr, and L must be greater then 0

def arrary is in meters (plotted in mm)
```

Below is a sample plot from simpbeam.m for the following data:

```
L = 0.10 m
I = 2.1333e-11 m^4   (4mm by 4mm shaft)
E = 200E+9 Pa  (steel)
incr = 100
P = [-100 100 -100] N
a = [0.02 0.05 0.08] m
```



On the deflection plot the red lines are at the point of force application and they are "pushing" the beam (e.i. a positive force will have its red line below the beam "pushing" up). The values of each force is displayed at the end of its force line.

The plot may look like the beam is bending a lot, but **compare at the scale on the x and y axis**. In this example the maximum deflection is **0.075 mm** for a 10 cm long beam!

Return to Top of Page

This site is maintained by rcortesi@mit.edu;
Send questions, comments, and suggestions to him.